

BUILDING ECONOMICS LIFE CYCLE COSTING

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CASH FLOW DIAGRAMS

During the lectures on Life Cycle Costing I will be sketching a cash flow diagram on the whiteboard before starting out on any calculations. I recommend students to always sketch a cash flow diagram before starting on their own Life Cycle Costing calculations or entry of values into a spreadsheet.

TIME PREFERENCE

- Economists term the observation that people seek a return to compensate for saving – forgoing consumption – an attitude of time preference.
- The term preference relates to what people would like to see occur rather than what they actually receive.
- According to Fisher (1930) the time preference of an individual will vary with six different factors:
 1. foresight
 2. self-control
 3. habit
 4. prospective length and certainty of life
 5. love of offspring and regard for posterity
 6. fashion
- A person with a high time preference seeks relatively more for abstinence than a person with a low time preference.

OPPORTUNITY COST

- Embodied in the attitude of time preference is the concept that to obtain the preferred rate of return the investor gives up an alternative.
- To save, you sacrifice consumption. To invest in a hydro dam the nation gives up additional roading, and so on. This concept is a method of evaluation called opportunity cost – a thing is valued by the next best alternative sacrificed to obtain it.

DISCOUNT RATE

- Discounting of future income is the process of converting future income into an equivalent present value. This process takes a future dollar amount and reduces it by a discount factor that reflects the appropriate interest rate. The rate at which future incomes are discounted is called the discount rate.
- The interest rate that induces a person to forgo expenditure today in order to have money to spend at some future date is a measure of that person's discount rate. Similarly people also borrow plus an interest charge for the use of the money. This interest rate also measures people's discount rate.
- The discount rate used to establish the net present value of future services depends on the opportunity costs for that class of investor. The factors involved include risk and the social

discount rate.

- When a selected discount rate is high the present value of distant future services are heavily discounted so that the returns on capital are expected to be realised within a shorter time frame.

SOCIAL DISCOUNT RATE

- The social discount rate is based on society's collective rate of time preference.
- Lind (1982) argues that in an ideal economy that satisfies the conditions of perfect competition in a world of certainty the social discount rate is equal to the market rate of interest. In an ideal economy the market discount rate should therefore be used as the discount rate for evaluating public as well as private investment decisions.
- Pigou (1920) contends that Government is the guardian of the interests of future generations, as well as the interests of present, and therefore should use a social discount rate that is lower than the market rate of interest.
- Marglin (1963) has demonstrated that the social discount rate is smaller than the private discount rate.
- "... the community in its collective, political capacity properly sees to it – directly or indirectly – that investment opportunities with future returns too low to justify private exploitation without the intervention of the state are in fact undertaken". (Marglin, 1963)

SELECTION OF PRIVATE DISCOUNT RATE

- Choosing the right discount rate depends upon the circumstances and objectives of the investor. In most cases the choice depends upon whether the investor is financing the project through borrowed money, or from capital assets.
- When the project is financed through borrowed money, an appropriate discount rate would be the ruling interest rate or the rate at which the investor can borrow funds.
- If part of the capital is to be provided by the investor by way of equity funding, the appropriate discount rate to be applied to these funds is determined by the current and future rate of return for that investor's particular industry and, ultimately, by the best alternative use of such funds.
- Consideration must be given to the value of the internal rate of private funds, since they would be no longer available for other uses that would usually be at higher rates of return than for borrowed funds.
- Interest rate shifts may occur at any time during the entire economic process. Although these shifts can be anticipated, a degree of speculation is always involved at the beginning of an economic process and speculation ceases only upon termination of the process (Bon, 1990).

ESCALATION

- Future prices can be expected to increase and this must be taken into account in any economic analysis. Escalation rates are particular to the cost item in question and can usually be expected to differ for each category.
- The inflation rate is the percentage annual increase in a general price level, commonly measured by the consumer price index (CPI) or some comparable price index.

REAL RATES OF INTEREST

- The real rate of interest is a measure of the relationship between escalation and interest rates.
- Nominal interest rates on the average are generally greater than escalation over the long run allowing the investor to earn an amount that compensates for escalation (inflation) plus an extra amount that is termed the "real rate of return".
- Historically, real interest rates are in the 2% to 5% range.
- The real rate of interest is related to the nominal rate of interest as follows:

value of \$1 after a one-year period at nominal interest = $1 + i$.

escalated value of \$1 after a one-year period = $1 + e$.

real value of \$1 after a one-year period

$$\frac{1+i}{1+e} = 1+R$$

where R is termed the real rate of interest.

$$R = \frac{1+i}{1+e} - 1$$

- If you were to invest \$100 now at $i = 10\%$ to pay for something costing \$100 now, but escalating at a rate of $e = 8\%$, in a years time you would have \$110 to buy an item that will cost \$108. The difference of \$2 represents the real amount earned and since the real rate of return must be based on the escalated value of the item, namely \$108, it is equal to the following:

$$R = \frac{2}{108} = 1.85\%$$

$$\text{or } R = \frac{1+i}{1+e} - 1 = \frac{1+0.10}{1+0.08} - 1 = 0.0185 = 1.85\%$$

- As an approximation, the difference between the interest rate and the escalation rate, which is termed the spread (S), may be assumed to be equal to the real rate of interest

$$S = i - e = 10\% - 8\% = 2\%$$

SINGLE PAYMENT COMPOUND AMOUNT FACTOR: SPCAF

If a present sum is invested today at a compound rate of interest, what will its future sum be at the end of a certain period of time?

P = present sum

n = number of periods

i = interest rate

F = future sum at end of n periods

$$F_1 = P + Pi = P(I+i)$$

$$F_2 = P(I+i) + P(I+i)i = P(I+i)^2 \quad (1)$$

$$F_n = P(I+i)^n$$

- The bracketed term $(1 + i)^n$ may be replaced by its abbreviation, SPCAF, single payment compound amount factor for n periods, and the above equation becomes

$$F_n = P \cdot SPCAF_i^n \quad (2)$$

SINGLE PAYMENT PRESENT WORTH FACTOR: SPPWF

A present sum of \$100 in hand is worth more than a future sum of \$100 in 5 years time, given a positive real rate of interest, because that present sum can be invested and accrue compound interest which will result in a future sum greater than \$100.

- A future sum is worth less than the same present sum. The present value of a future sum can be found by discounting, which is the same as compounding performed in reverse:

$$F = \frac{S}{(I+i)^n} \quad (3)$$

- The term $1/(1 + i)^n$ can be replaced by its abbreviation, SPPWF, single payment present worth factor for n periods. The above equation becomes

$$P = F \cdot SPPWF_i^n \quad (4)$$

- SPPWF is also known as the Discount Factor.

UNIFORM SERIES COMPOUND AMOUNT FACTOR: USCAF (FUTURE VALUE OF ANNUITY)

Given a uniform series of end of period payments (A), what will n payments accumulate to at compound interest i ?

$$F = A(1+i)^{n-1} + A(1+i)^{n-2} \dots A(1+i) + A \quad (5)$$

- Multiply above equation by $(1 + i)$

$$F(1+i) = A(1+i)^n \dots A(1+i)^2 + A(1+i) \quad (6)$$

- Subtract first equation from above

$$F(1+i) - F = A(1+i)^n - A \quad (7)$$

$$F = A \left[\frac{(1+i)^n - 1}{i} \right]$$

- The right hand expression in square brackets can be replaced by its abbreviation USCAF, uniform series compound amount factor. The above equation becomes

$$F = A \cdot USCAF_i^n \quad (8)$$

SINKING FUND DEPOSIT FACTOR: SFDF (SINKING FUND FACTOR)

The previous equation allows us to compute A , the uniform series of end-of-period deposits that must be made for n periods at compound interest i in order to provide a required sum F (sinking fund).

$$A = F \left[\frac{i}{(1+i)^n - 1} \right] = F \cdot SFDF_i^n \quad (9)$$

- The expression in square brackets is abbreviated by SFDF, sinking fund deposit factor.

UNIFORM SERIES CAPITAL RECOVERY FACTOR: CRF (INSTALMENT TO AMORTISE)

Capital recovery requires not only the repayment of capital by some future date but also the payment of interest for the use of the capital during the period concerned.

- The capital recovery equation enables us to compute for A, a uniform series of end-of-period payments that will just recover a present sum P over n periods with compound interest i.
- Substitute the expression for F from Equation (1) into Equation (9) to express A in terms of P:

$$A = P(1+i)^n \cdot \frac{i}{(1+i)^n - 1} = P \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right] = P \cdot CRF_i^n \quad (10)$$

- The expression in square brackets is abbreviated by CRF, capital recovery factor.

UNIFORM SERIES PRESENT WORTH FACTOR: USPWF (PRESENT VALUE OF AN ANNUITY)

The inversion of Equation (10) enables us to compute the present value (P) of a uniform series of payments (A).

$$P = A \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right] = A \cdot USPWF_i^n \quad (11)$$

- The expression in square brackets is abbreviated by USPWF, uniform series present worth factor.

ANNUAL PERCENTAGE RETURN: APR

- Unless specified otherwise, compounding is done annually using the nominal interest rate i and the period n years. Interest at 8 percent means 8 percent compounded annually.
- Interest at 8 percent quarterly still means an 8 percent nominal interest rate, but its annual percentage return, or actual interest rate, will be greater since n is now in quarters and compounding is done more frequently. The annual percentage return is the percentage increase in principal per annum.

$$\text{Annual percentage return} = \left(\frac{F - P}{P} \right) \cdot 100 = SPCAF_i^n - 1 \quad (12)$$

where n is the number of interest periods per year and i is the interest rate per period.

EFFECTIVE RATE OF INTEREST

The effective rate of interest is the rate per period which, when compounded, equates to the annual percentage rate (APR).

If r is the annual percentage rate and n is the number of periods per year, then the effective rate of interest for each period is given by:

$$\text{Effective rate of interest} = (1 + r)^{\frac{1}{n}} - 1 \quad (13)$$

For example, if the annual percentage rate is 12% and interest is charged quarterly, then the effective rate of interest each quarter is given by:

$$r_{\text{quarter}} = (1 + 0.12)^{\frac{1}{4}} - 1 = 0.02874 = 2.874\%$$

and not 3%

CONTINUOUS COMPOUNDING

If interest is compounded an infinite number of times a year, i.e. continuously, then the equation for the annual percentage return is derived as follows:

let r = nominal continuous interest rate, m = number of periods of compounding.

Then

$$SPCAF = \left(1 + \frac{r}{m}\right)^m = \left(1 + \frac{r}{m}\right)^{\frac{m}{r} \cdot r}$$

But

$$\lim_{m \rightarrow \infty} \left(1 + \frac{r}{m}\right)^{\frac{m}{r}} = e$$

$$\text{Annual percentage return} = e^r - 1 \quad (14)$$

CASH FLOW DIAGRAM CONVENTIONS

If cash flows include both income and costs, then income is shown as positive above the time line and costs are shown negative below the time line so as to avoid ambiguity.

If cash flows are all costs, then costs are shown above the timeline for convenience.

INVERSE RELATIONSHIP OF DISCOUNT FACTORS

The following discount factors have an inverse relationship. Have a look at the formulae and values in a discount table.

SPACAF and SPPWF

UDCAF and SFDF

USPWF and CRF

NEED FOR CONSISTENCY IN IDENTIFYING THE PERIOD AND THE DISCOUNT RATE

For example using the Capital Recovery Factor CRF for a period of 10 years and a discount factor of 10%:

$$CRF_{0.10}^{10} \quad CRF_{0.10}^{10}$$

The upper level value in superscript is the year 10

The lower level value on the line or subscript is the discount rate of 10% (0.10)

The discount rate can be written on one line without superscripts and subscripts as follows:

$$CRF(10,0.10)$$

The year in brackets is first and the discount rate is second. Consistency is paramount.

START OF FIRST CASH FLOW IN UNIFORM SERIES DISCOUNT TABLES

Note that in a standard table of discount factors the cash flow of a uniform series starts at the end of the first year. Have a look at year 1 of the USCAF, SFDF, USPWF, and CRF discount factors in the discount table you are using to confirm this.

EXAMPLES USING COMPOUND INTEREST FACTORS

EXAMPLE 1

What will \$100 now be worth in 10 years at 6 percent interest?

- The problem is to find F from P, given i and n.

$$F = P \cdot SPCAF_{0.06}^{10} = 100 \times 1.7908 = \$179.08$$

EXAMPLE 2

What is the present value of \$100 reached after 5 years at 8 percent nominal interest if (a) interest is compounded annually, (b) interest is compounded quarterly?

- The problem is to find P from F, given i and n.

$$i = 8 \text{ percent, } n = 5$$

$$P = F \cdot SPPWF_{0.08}^5 = 100 \times 0.6806 = \$68.06$$

$$i = 8/4 = 2 \text{ percent, } n = 5 \times 4 = 20$$

$$P = F \cdot SPPWF_{0.02}^{20} = 100 \times 0.6730 = \$67.30$$

EXAMPLE 3

If \$100 were put aside at the end of each year, what sum of money would be accumulated at the end of 15 years at 3 percent interest?

- The problem is to find F from A, given i and n.

$$F = A \cdot USCAF_{0.03}^{15} = 100 \times 18.599 = \$1859.90$$

EXAMPLE 4

How much should be saved at the end of each year to obtain a sum of \$10,000 at the end of 20 years (a) at 3 percent interest (b) at 6 percent interest?

- The problem is to find A from F, given i and n.

i = 3 percent, n = 20

$$A = F \cdot SFDF_{0.03}^{20} = 10,000 \times 0.03722 = \$372.20$$

i = 6 percent, n = 20

$$A = F \cdot SFDF_{0.06}^{20} = 10,000 \times 0.02718 = \$271.80$$

EXAMPLE 5

- What payment is required at the end of each six months in order to repay a mortgage of \$10,000 in 30 years at 6 percent nominal interest?
- The problem is to find A from P given i and n.

i = 6/2 = 3 percent, n = 30 x 2 = 60

$$A = P \cdot CRF_{0.03}^{60} = 10,000 \times 0.03613 = \$361.30$$

EXAMPLE 6

- How much can you justify spending now in order to save spending \$100 a year for 10 years if money is worth 8 percent?
- The problem is to find P from A, given i and n.

$$P = A \cdot USPWF_{0.08}^{10} = 100 \times 6.710 = \$671$$

EXAMPLE 7

- What is the annual percentage return if the nominal interest rate is 12 percent and interest is compounded monthly?
- $i = 12/12 = 1$ percent, $n = 1 \times 12 = 12$

$$SPCAF_{0.01}^{12} - 1 = 1.1268 - 1 = 0.1268 = 12.68 \text{ percent}$$

EXAMPLE 8

- At what interest rate would money double itself in 5 years?
- The problem is to find i , given n and the ratio of S and P .

$$2P = P \cdot SPCAF_i^5 \quad \text{or} \quad SPCAF_i^5 = 2$$

at 12 percent, $SPCAF = 1.7623$

at 15 percent, $SPCAF = 2.0114$

- By linear interpolation

$$i = 12 + (15 - 12) \times \frac{(2 - 1.7623)}{(2.0114 - 1.7623)} = 12 + 2.86 = 14.86 \text{ percent}$$

A rule of thumb is that if a population is growing exponentially, then the duration required to double in size is 72 (or 70 for an approximate answer) divided by the continuous rate of growth. In this case the duration to double would be $72/5 = 14.40$ years.

EXAMPLE 9

- What is the perpetual annual income or annuity that \$10,000 can purchase if interest is at 3 percent?
- The problem is to find A , being given P , i and n (infinity).

$$CRF = \frac{i(1+i)^n}{(1+i)^n - 1} = \frac{i(1+i)^n - i + i}{(1+i)^n - 1} = \frac{i[(1+i)^n - 1] + i}{(1+i)^n - 1} = i + \frac{i}{(1+i)^n - 1}$$

- When n approaches infinity

$$CRF = i + \frac{i}{(1+i)^\infty - 1} = i + 0 = i$$

- A perpetual income can come from only interest, leaving the capital untouched.

$$R = 10,000 \times 0.03 = \$300$$

Intuitively, if you donate \$100,000 to a trust account and you want that trust to deliver a constant income into perpetuity, then the trust must not draw down on that capital. The constant income that can be drawn down is the interest on the capital.

USING REAL OR NOMINAL DISCOUNT RATES

The results are the same whether real or nominal discount rates are used. Calculations are easier when real discount rates are used even though discount rates are normally given as nominal rates. Using real discount rates is a distinct advantage when inflation rates for different items are not the same.

EXAMPLE

Find the present value PV of a uniform series of three annual \$1,000 real term payments when the nominal discount rate is 15% and inflation is running at 5% per annum.

Using a real discount rate:

The discount rate and payments are in real terms

The real discount rate is approximately 15% - 5% = 10%

$$PV = 1,000 \times USPWF_{0.10}^3 = 1,000 \times 2.486852 = \$2,486.85$$

Using a nominal discount rate:

The discount rate and payments are in nominal terms. The first payment is \$1,000 but inflation is running at 5% therefore future payments are not uniform and need to be escalated as follows:

$$\text{Year 1 payment } PV = 1,000 \times \text{SPCAF}_{0.05}^1 = 1,000 \times 1.05 = \$1,050$$

$$\text{Year 2 payment } PV = 1,000 \times \text{SPCAF}_{0.05}^2 = 1,000 \times 1.1025 = \$1,102.50$$

$$\text{Year 3 payment } PV = 1,000 \times \text{SPCAF}_{0.05}^3 = 1,000 \times 1.157625 = \$1,157.25$$

The present value PV is the sum of the following:

$$PV_1 = 1,050.00 \times \text{SPPW}_{0.15}^1 = 1,050.00 \times 0.869565 = \$913.04$$

$$PV_2 = 1,102.50 \times \text{SPPW}_{0.15}^2 = 1,102.50 \times 0.756144 = \$833.65$$

$$PV_3 = 1,157.63 \times \text{SPPW}_{0.15}^3 = 1,157.25 \times 0.657516 = \$760.91$$

$$PV = \$913.04 + \$833.75 + \$760.91 = \$2,507.60$$

The difference of the real PV from the nominal PV is -0.83%. In other words the approximate real discount rate of 10% is too high.

The more accurate discount rate using discount tables is as follows:

$$R = \frac{1+0.15}{1+0.05} - 1 = \frac{1.15}{1.05} - 1 = 9.52\%$$

Use the real discount rate of 9.52% in the following calculation:

$$PV = 1,000 \times \text{USPWF}_{0.0952}^3 = 1,000 \times 2.50802 = \$2,508.02$$

The two values of the present value PV would be exactly the same if a higher degree of precision for the real discount rate R was used. An electronic spreadsheet would provide a higher degree of accuracy than a hand calculator making use of discount tables.

INVESTMENT APPRAISAL TECHNIQUES

Any acceptable investment appraisal technique must exhibit two properties:

- It should take account of all cash flows associated with the investment throughout the period of analysis.
- It must make proper allowance for the time value of money.

There is a third criterion that should also be required of an investment:

- The return on the investment should not be less than the market rate of interest after allowing for risk.

ANNUAL EQUIVALENT VALUE: AEV (EQUIVALENT ANNUAL COST, COST IN USE)

DESCRIPTION OF AEV

- The annual equivalent value is the regular annual cost (annuity) which, when discounted, equals the Net Present Value (NPV) of the investment.
- The annual equivalent method should be used only when there is a natural replacement cycle over which the component or finish is completely replaced; it does not refer to a regular maintenance cycle.
- There is an implicit assumption that the same component or finish will be used in each replacement cycle.

METHOD OF AEV

AEV = Annual Equivalent Value

NPV = Net Present Value

CRF = Capital Recovery Factor

$$AEV = NPV \cdot CRF$$

SELECTION CRITERIA OF AEV

- Select the investment with the lowest Annual Equivalent Value (AEV) in order to minimise costs.

CLASS EXERCISE

If you have a stream of \$1,000 each year for 5 years starting at the end of year 1, what is the Annual Equivalent Value (AEV) of this stream?

$$AEV = NPV \cdot CRF = (A \cdot USPWF) \cdot CRF = A$$

The Annual Equivalent Value of a regular stream of identical cash flows starting at the end of the year one is the value of the first cash flow. This also applies for a stream of cash flows that continues into perpetuity.

CLASS EXERCISE

What is the Present Value (PV) of a perpetual stream of payments of \$12,000 using a 12% discount rate?

$$AEV = A = NPV \cdot CRF$$

The Capital Recovery Factor $CRF =$ the discount rate i when the period n approaches infinity.

$$A = NPV \cdot i \Rightarrow PV = \frac{A}{i}$$
$$PV = \frac{12,000}{0.12} = \$100,000$$

CLASS EXERCISE

What is the PV of a perpetual stream of payments of \$12,000, 12% discount rate, which starts at the end of the third year?

First calculate the Present Value of the perpetual stream at the end of the second year.

$$PV_{(2)} = \frac{A}{i} = \frac{12,000}{0.12} = \$100,000$$

Next calculate the Present Value at the start of the first year

$$PV_{(0)} = PV = F \cdot SPPWF_{0.12}^2 = 100,000 \times 0.797194 = \$79,719.40$$

CLASS EXERCISE – DEMONSTRATION OF EQUIVALENCE

Calculate the PV of a regular stream of payments of \$100 starting at the end of year 1 and finishing at the end of year 5 by

- Adding up the PV of each individual payment
- Adding up the individual Future Value (FV) of each individual payment at the end of year 5 and then calculating the PV of that summed FV.

First method a:

$$PV \text{ of } FV \text{ at end of year 1} = F_{(1)} \cdot SPPWF_{0.10}^1 = 100 \times 0.909091 = \$90.90$$

$$PV \text{ of } FV \text{ at end of year 2} = F_{(1)} \cdot SPPWF_{0.10}^2 = 100 \times 0.826446 = \$82.64$$

$$PV \text{ of } FV \text{ at end of year 3} = F_{(1)} \cdot SPPWF_{0.10}^3 = 100 \times 0.751315 = \$75.13$$

$$PV \text{ of } FV \text{ at end of year 4} = F_{(1)} \cdot SPPWF_{0.10}^4 = 100 \times 0.683013 = \$68.30$$

$$PV \text{ of } FV \text{ at end of year 5} = F_{(1)} \cdot SPPWF_{0.10}^5 = 100 \times 0.620921 = \$62.90$$

$$\text{Sum of } PV = \$379.08$$

Second method b:

$$FV \text{ of } PV \text{ at end of year 1} = P_{(1)} \cdot SPCAF_{0.10}^4 = 100 \times 1.4641 = \$146.41$$

$$FV \text{ of } PV \text{ at end of year 2} = P_{(2)} \cdot SPCAF_{0.10}^3 = 100 \times 1.3310 = \$133.10$$

$$FV \text{ of } PV \text{ at end of year 3} = P_{(3)} \cdot SPCAF_{0.10}^2 = 100 \times 1.2100 = \$121.00$$

$$FV \text{ of } PV \text{ at end of year 4} = P_{(4)} \cdot SPCAF_{0.10}^1 = 100 \times 1.100 = \$110.00$$

$$FV \text{ of } PV \text{ at end of year 5} = P_{(5)} \cdot SPCAF_{0.10}^0 = 100 \times 1.0000 = \$100.00$$

Each payment earns compound interest, i.e. earns its own keep.

$$\text{Sum of } PV = FV \cdot SPPWF_{0.10}^5 = 610.51 \times 0.620921 = \$379.08$$

The two different methods give the same result

EXAMPLE 1

Choose between the alternatives of having \$2000 now or \$156 per annum for 16 years if the nominal interest rate is 6 percent.

- Alternative (A) $P = \$2000$, $n = 16$, $i = 6$ percent.
- Alternative (B) $A = \$156$ p.a. for 16 years, $i = 6$ percent.

When $i = 6$ percent

$$CRF_i^n = 0.09895$$

$$A = P \cdot CRF$$

$$2000 \times 0.09895 = \$197.90 > \$156$$

- Alternative A is favoured.

EXAMPLE 2

A firm is faced with the problem of choosing from two machines A and B, both capable of doing the same work. A 10 percent minimum nominal rate of return is required.

If no inflation rate is included, then we can assume that inflation is zero i.e. the nominal discount rate is equal to the real discount rate.

	Machine A	Machine B
First Cost	\$20,000	\$14,000
Estimated Economic Life	10 years	10 years
Salvage Value	Nil	Nil
Annual Running Cost	\$2000	\$3000

- Machine A

$$\begin{aligned} AEV &= 20,000 \times UCRF_{0.10}^{10} + 2000 \\ &= 20,000 \times 0.16275 + 2000 = \$5255 \end{aligned}$$

- Machine B

$$AEV = 14,000 \times 0.16275 + 3000 = \$5280$$

- Machine A has a lower Annual Equivalent Value (Cost) by \$25.

EXAMPLE 3

A firm is faced with the problem of choosing from two machines C and D with different economic lives, both capable of doing the same work. A 10 percent minimum nominal rate of return is required.

	Machine C	Machine D
First Cost	\$12,000	\$40,000
Estimated Economic Life	10 years	20 years
Salvage Value	Nil	\$10,000
Annual Running Cost	\$2500	\$1000

Machine C

$$\begin{aligned}
 AEV &= 12,000 \times UCRF_{0.10}^{10} + 2500 \\
 &= 12,000 \times 0.16275 + 2500 = \$4450
 \end{aligned}$$

- Machine D

$$\begin{aligned}
 AEV &= (40,000 - 10,000) \times UCRF_{0.10}^{20} + 10,000 \times 0.1 + 1000 \\
 &= 30,000 \times 0.11746 + 1000 + 1000 = \$5520
 \end{aligned}$$

- Capital recovery of \$30,000 is provided but interest only is paid on the salvage value of \$10,000.

EXAMPLE 4

A firm is faced with the problem of choosing from two machines E and F, both capable of doing the same work. A 20 percent minimum nominal rate of return is required.

	Machine E	Machine F
First Cost	\$20,000	\$30,000
Annual Operating Cost	\$3000	\$1500
Major Overhaul Cost	\$5000	\$3000
Due at end of	8 years	10 years
Economic Life	15 years	15 years
Salvage Value	\$3000	\$5000

Machine E

$$\begin{aligned}AEV &= (20,000 + 5,000 \times SPPWF_{0.20}^8 - 3,000 \times SPPWF_{0.20}^{15}) \times UCRF_{0.20}^{15} + 3,000 \\ &= (20,000 + 5,000 \times 0.2326 - 3,000 \times 0.0649) \times 0.21388 + 3,000 = \$7,485\end{aligned}$$

- Machine F

$$\begin{aligned}AEV &= (30,000 + 3,000 \times SPPWF_{0.20}^{10} - 5,000 \times SPPWF_{0.20}^{15}) \times UCRF_{0.20}^{15} + 1,500 \\ &= (30,000 + 3,000 \times 0.1615 - 5,000 \times 0.0649) \times 0.21388 + 1,500 = \$7,950\end{aligned}$$

- There will be an annual saving of \$466 if Equipment E is installed instead of F.

NON-UNIFORM SERIES OF OPERATING AND MAINTENANCE COSTS

- Convert all sums to their present worths and then convert the sum of these present worths to a uniform series.

ADVANTAGES OF AEV

- The Annual Equivalent Value (AEV) method provides a short cut for comparing investments with different life cycles.

CLASS EXERCISE – CAPITAL RECOVERY

You have a capital expenditure of \$100 at year zero and an income stream of \$10 each year for 5 years starting at the end of year 1. The discount rate is 10%. Calculate the NPV.

$$PV_{(\text{income})} = A \cdot USPWF_{0.10}^5 = 10 \times 3.790787 = \$37.91$$

$$NPV = \$37.91 - \$100 = -\$62.09 \text{ i.e. negative or } (\$62.09) \text{ using accounting format}$$

This time around you have the same above capital expenditure and income and the same discount rate but at the end of year 5 you have an additional income of \$100 where the salvage value is the same as the capital expenditure (unlikely in practice). Calculate the NPV

$$PV_{(\text{income})} = 10 \times USPWF_{0.10}^5 + 100 \times SPPWF_{0.10}^5 = 10 \times 3.790787 + 100 \times 0.62092 = \$37.91 + \$62.09 = \$100$$

$$NPV = \$100 - \$100 = 0$$

The yield of the cash flows is 10% (the Internal Rate of Return when NPV = 0) and include the cash flow includes recovery of the capital expenditure.

What uniform series amount will accumulate to \$100 by the end of year 5?

$$A = F \times SFDP_{0.10}^5 = 100 \times 0.163797 = \$16.38$$

Now add \$16.38 to the annual flow of \$10 each for 5 years starting at the end of year 1.

You have a capital expenditure of \$100 at year zero and an income stream of \$26.38 each year for 5 years starting at the end of year 1. The discount rate is 10%. Calculate the NPV.

$$PV_{(\text{Income})} = A \cdot USPWF_{0.10}^5 = 26.38 \times 3.790787 = \$100$$

$$NPV = \$100 - \$100 = 0$$

Capital recovery of an initial investment can occur through increased flows of income if the salvage value is zero.

Alternatively calculate the uniform series of income for 5 years starting at the end of year 1 which will equal a PV of \$100 when the discount rate is 10%.

$$AEV = PV \cdot CRF_{0.10}^5 = 100 \times 0.263797 = \$26.38$$

CLASS EXERCISE – AVERAGE WEIGHTED RETURN

Consider a uniform series of income of \$12 for 4 years starting at the end of year 1 and an income of \$112 at the end of year 5. The capital expenditure at the start of year 1 is \$100. The discount rate is 12%. Calculate the NPV.

$$PV_{(\text{Income})} = 12 \times USPWF_{0.12}^5 + 100 \times SPPWF_{0.12}^5 = 12 \times 3.60477 + 100 \times 0.567427 = \$43.26 + \$56.74 = \$100$$

$$NPV = \$100 - \$100 = 0$$

As in the previous class exercise demonstrating capital recover, the NPV is zero. The yield of the process is 12% (IRR = 12%).

Using the same above example, for capital expenditure at the start of year 1 borrow 50% at 8% and use 50 % equity at 12%.

$$\text{Average Weighted Return} = 8\% \times \frac{50}{50+50} + 12\% \times \frac{50}{50+50} = 6\% + 4\% = 10\%$$

Now discount the cash flows at 10%:

$$PV_{(\text{Income})} = 12 \times USPWF_{0.10}^5 + 100 \times SPPWF0 = 12 \times 3.790787 + 100 \times 0.620921 = \$45.49 + \$62.09 = \$107.58$$

$$NPV = \$107.58 - \$100 = \$7.58$$

The NPV is positive to the value of \$7.58 as a result of borrowing 50% of the capital expenditure at a lower interest rate of 8% instead of using 100% equity at a discount rate of 12%.

Refer to page 5 of the Building Economics Lecture – Economic Evaluation in Practice part of which is repeated here:

- If the interest rate and discount rate are not identical then the life cycle implications of the capital plus interest charges can be calculated by:

(1) Calculating the value of an annuity of $(i - r)C_0$ at interest rate r over N years. (i = interest rate on the borrowed capital, r = discount rate.)

(2) Adding the result of (1) to the initial capital costs.

If $i > r$, then interest charges increase life cycle costs. If $i < r$, then interest charges decrease life cycle costs

Now calculate the value of the annuity:

$$(i - r) \cdot C_0 = (8\% - 12\%) \times \$50 = -4\% \text{ of } \$50 = -\$2 \text{ of } (\$2) \text{ in accounting format}$$

Now calculate the PV of the annuity of \$2 over five years discounted at 12%:

$$PV_{(\text{Annuity})} = -\$2 \times USPWF_{0.12}^5 = -\$2 \times 3.60477 = -\$7.21 \text{ or } (\$7.21) \text{ in accounting format}$$

The interest rate on the borrowed capital is less than the discount rate, therefore the result of borrowing 50% of the capital expenditure at the start of year 1 is the same as reducing the capital expenditure by \$7.21. The capital expenditure would be $\$100 - \$7.21 = \$92.79$.

Consider a uniform series of income of \$12 for 4 years starting at the end of year 1 and an income of \$112 at the end of year 5. The capital expenditure at the start of year 1 is \$92.79. The discount rate is 12%. Calculate the NPV.

$$PV_{(\text{Income})} = 12 \times USPWF_{0.12}^5 + 100 \times SPPWF_{0.12}^5 = 12 \times 3.60477 + 100 \times 0.567427 = \$43.26 + \$56.74 = \$100$$

$$NPV = \$100 - \$92.79 = \$7.21$$

The NPV of \$7.21 is approximately the same as \$7.58 using the Weighted Average Returns (WAR) method. The difference is because the WAR method is based on the simplifying assumption of cash flows into perpetuity.

CLASS EXERCISE – DANGERS OF USING IRR FOR DECISION MAKING

The Internal Rate of return of the above cash flows using a financial calculator or spreadsheet is 14.11% compared to the discount rate of 12%. This is due to the leverage of borrowing 50% of the capital costs at a lower interest rate.

Now consider the cash flows of the equity. The income is \$12 each year for 5 years plus an additional income of \$100 at the end of year 5, but interest of \$4 each year is paid on the borrowed \$50 (8% of \$50 = \$4). The net income on equity is therefore \$8 per year. At the end of year 5 the net income of \$108 is further reduced by \$50 which is used to repay the loan.

You have a uniform series of income of \$8 for 4 years starting at the end of year 1 and an income of \$58 at the end of year 5. The capital expenditure at the start of year 1 is \$50. The discount rate is 12%. Calculate the NPV.

$$PV_{(\text{Income})} = 8 \times USPWF_{0.12}^5 + 50 \times SPPWF_{0.12}^5 = 8 \times 3.60477 + 50 \times 0.567427 = \$28.84 + \$28.37 = \$57.21$$

$$NPV = \$57.21 - \$50 = \$7.21$$

The NPV is the same as before, but the IRR of the cash flows is 16.0%

It is safer to strike a discount rate and estimate the NPV
than to use the IRR of cash flows as a basis for decision making.

If the capital expenditure is 100% financed using a loan at 8% then the net income is \$8 each year (8% of \$100 = \$8) for five years.

You have a uniform series of income of \$4 for 4 years starting at the end of year 1 and an income of \$4 at the end of year 5. The capital expenditure at the start of year 1 is \$0. The discount rate is 12%. Calculate the NPV.

$$NPV = \$4 \times USPWF_{0.12}^5 = \$4 \times 3.60477 = \$14.42 \text{ which is far greater than } \$7.21$$

The IRR of the above cash flow approaches infinity. It is nonsense to talk about the IRR of an investment if there is no equity investment and there is 100% gearing.

CLASS EXERCISE – DIFFERENT ECONOMIC LIVES

Please note the following values used are for demonstration purposes only.

Building component A costs \$10,000, an annual maintenance cost of \$50, and a service life of 40 years. Building component B costs \$7,000, an annual maintenance cost of \$300, and a service life of 20 years. The discount rate is 10%. Calculate the Present Value Costs PVC.

$$PVC_A = 10,000 + 50 \times USPWF_{0.10}^{40} = 10,000 + 50 \times 9.779 = 10,000 + 488.95 = \$10,488.95$$

We need to compare the two components over the same time frame, so we calculate for two cycles of 20 years = 40 years for component B.

$$PVC_A = 7,000 + 7,000 \times SPPWF_{0.10}^{20} + 300 \times USPWF_{0.10}^{40} = 7,000 + 7,000 \times 0.1486 + 300 \times 9.779 = \$10,973.90$$

Component B is \$484.95 more expensive than component A in PV terms.

Now consider the difference in cost in AEC terms:

$$\Delta AEV_{AB} = \Delta PV_{AB} \cdot CRF_{0.10}^{40} = 484.95 \times 0.102259 = \$49.49$$

Now consider the AEV of component B over two cycle where PVC = \$10,973.90

$$AEV_B = 10,973.90 \times CRF_{0.10}^{40} = 10,973.90 \times 0.102259 = \$1,122.18$$

Now consider the AEV of component B over one cycle:

$$AEV_B = 7,000 \times CRF_{0.10}^{20} + 300 = 7,000 \times 0.117460 + 300 = \$1,122.18$$

When calculating the AEV of a building component, then we need to do so over only once cycle provided each subsequent cycle is identical in real costs i.e. no escalation. There is an assumption that components are replaced with exactly the same component. This is not the case with machinery where subsequent replacements have undergone technological improvements and have different operating costs.

Note that a life cycle includes salvage of the previous component and that installation of the first component does not include this cost. Comparisons of the AEV of building components are comparing the AEV of subsequent replacements.

NET PRESENT VALUE (NPV)

DESCRIPTION OF NPV

- Net Present Value (NPV) is the sum of present values of a stream of future costs and benefits of an investment.

METHOD OF NPV

- All benefits and costs during the period of analysis are converted to present values, i.e. benefits and costs in terms of a baseline dollar, usually today's dollar.
- Savings are regarded as benefits.
- Benefits are positive inflows, costs are negative outflows.
- The present values of all benefits and costs are summed to produce the Net Present value.
- The procedure of summing up the present value of a time series of monetary amounts is usually referred to as Discounted Cash Flow (DCF).

- Given

B_t = estimated benefits in year t

C_t = estimated cost in year t

d = discount rate

T = period of analysis of T years

The Net Present Value of an investment is

$$\begin{aligned}
 NPV &= B_0 + \frac{B_1}{1+d} + \dots + \frac{B_t}{(1+d)^t} + \dots + \frac{B_T}{(1+d)^T} - C_0 - \frac{C_1}{(1+d)} - \dots - \frac{C_t}{(1+d)^t} - \dots - \frac{C_T}{(1+d)^T} \\
 &= \sum_{t=0}^T \frac{B_t}{(1+d)^t} - \sum_{t=0}^T \frac{C_t}{(1+d)^t} = \sum_{t=0}^T \frac{B_t - C_t}{(1+d)^t}
 \end{aligned}$$

where $B_t - C_t$ is the Net Return received in any year t.

- When costs only are involved, NPV is the value of a sinking fund that would have to be established today in order to cover all of the cash outflows associated with the project.

SELECTION CRITERIA OF NPV

- Any project with a non-negative NPV meets the test at the test discount rate (d).
- In the case of competing projects those with the highest NPV should be selected in descending order of NPVs until the budget is exhausted.

EXAMPLE 1

Two schemes of water storage are being examined. Both are capable of satisfying the service requirement. Scheme A costs \$200,000 to build and has an estimated annual running cost of \$30,000. Scheme B costs \$100,000 more to build but \$9,000 a year less to operate. Both schemes are assumed to have an economic life of 40 years. Nominal interest rate is 6 percent. Which scheme should be recommended?

- Scheme A

$$\begin{aligned}
 NPV_A &= 200,000 + 30,000 \times USPWF_{0.06}^{40} \\
 &= 200,000 + 30,000 \times 15.046 = \$650,000
 \end{aligned}$$

- Scheme B

$$NPV_B = 300,000 + 21,000 \times 15.046 = \$615,000$$

- The Net Present Value (Cost) of scheme B is less than scheme A.

EXAMPLE 2

A section of highway at present costs \$1000 a year to maintain. A new pavement is proposed which will reduce the maintenance cost by \$800 a year during the first five years, by \$500 a year during the next 15 years and by nothing thereafter. How much capital cost can be justified on the new pavement at a nominal discount rate of 6 percent?

- Present worth of savings

$$\begin{aligned} P &= 800 \times USPWF_{0.06}^5 + 500 \times USPWF_{0.06}^{15} \times SPPWF_{0.06}^5 \\ &= 800 \times 4.2124 + 500 \times 9.7122 \times 0.74726 = \$7000 \end{aligned}$$

- At a discount rate of 6 percent, an expenditure of no more than \$7000 can be justified on the new pavement.
- Note that the second series has to be discounted twice. The first factor, USPWF, brings it to its 'present worth' five years from now; the second factor, SPPWF, brings it to its true present worth.

EXAMPLE 3

A government stock with a face value of \$1000 and bearing 4 percent interest, payable semi-annually, has 14 years to go before maturity. What is the value of the stock if you wish a nominal return of 8 percent on your investment?

- Interest payable 6-monthly = $0.02 \times 1000 = \$20$
Number of period left = $14 \times 2 = 28$
Lump sum of \$1000 available at end of 14 years

- Present Worth of stock

$$\begin{aligned} P &= 20 \times USPWF_{0.04}^{28} + 1000 \times SPPWF_{0.08}^{14} \\ &= 20 \times 16.33 + 1000 \times 0.4423 = \$769 \end{aligned}$$

EXAMPLE 4

- The details of one of the alternatives being considered in a study are as follows

Capital investment	= \$10,000
Real interest rate (i)	= 10%
Real escalation rates (e)	= nil for maintenance
(Note example rates are not realistic values)	= 4% for materials = 9% for energy
Time period	= 20 years
Annual maintenance	= \$400
Component replacement cost in 10 years	= \$2000 at current cost
Annual energy costs	= \$500 at current cost
Residual value (20 years)	= \$1000 at current cost

When escalation needs to be taken into account we can simplify our calculations by using Geometric Series Present Worth Factor (GSPWF) tables or the formula as follows:

Real discount rate = i , real escalation rate = e

$$\text{For } i \neq e \quad P = A \cdot \left[\frac{1 - (1+e)^n \cdot (1+i)^{-n}}{i - e} \right]$$

$$\text{For } i = e \quad P = A \cdot \left[\frac{n}{1+i} \right]$$

To determine the Net Present Value

Present value of capital investment	\$10,000.00
Present Value of Maintenance	
400 x USPWF = 400 x 8.5135	\$3,405.40
(i = 10%, e = 0%, n = 20)	
Present value of component replacement in 10 years	
2000 x SPCAF x SPPWF	
= 2000 x 1.4802 x 0.3855	\$1,141.23
(i = 10%, e = 4%, n = 10)	
Present value of annual energy costs	
500 x GSPF = 500 x 18.20	\$9,100.00
(i = 10%, e = 9%, n = 20)	
Less present value of residual value	
1000 x SPCAF x SPPWF	

$$= 1000 \times 2.1911 \times 0.1486 \quad (\$325.60)$$

$$(i = 10\%, e = 4\%, n = 20)$$

$$\text{Net Present Value} \quad \$23,321.03$$

EXAMPLE 5 – EARLY REPLACEMENT OF HVAC

A building has a heating, ventilation, and air conditioning unit (HVAC) which could be used for two more years at the most. The HVAC unit original cost \$45,000 five years ago. Its salvage value is currently \$8,000 owing to a special opportunity for sale, but it would be zero at all subsequent times. If the HVAC unit were to be used for two more years, it would require a major overhaul at a cost of \$9,000 at the end of one year.

A new model of the HVAC unit is being marketed. It costs \$40,000 and has a maximum life of 10 year provide that special maintenance is undertaken at a cost of \$10,000 after five years and at a cost of \$20,000 after 8 years. Assume that the new model would have no salvage value at any time, no other new models are expected to become available in the foreseeable future, and that the running costs and periodic maintenance costs of the existing and new HVAC units are the same. Also assume that each new HVAC will be replaced upon the end of its life by an identical HVAC unit (a simplifying assumption). The building owner's cost of capital is 15% per annum.

Prepare calculations to show whether the existing HVAC unit should be replaced immediately or after one or two years.

The cost of the existing HVAC unit (\$45,000) five years ago is a sunk cost and therefore forms no part of any economic decision. Salvage values are always taken into account.

First consider the cash flow of the new model - \$40,000 at the start of year1, \$10,000 at the end of year 5, \$20,000 at the end of year 8, and a service life of 10 years with a zero salvage value. Because the running costs and periodic maintenance costs of the existing and new HVAC units are the same, these costs can be ignored.

$$\begin{aligned} PV_{\text{New one cycle}} &= 40,000 + 10,000 \times SPPWF_{0.15}^5 + 20,000 \times SPPWF_{0.15}^8 \\ &= 40,000 + 10,000 \times 0.497188 + 20,000 \times 0.326902 \\ &= \$51,509.81 \end{aligned}$$

The HVAC unit will be replaced after 10 years:

$$AEV_{\text{New, one cycle}} = 51,509.81 \times CRF_{0.15}^5 = 51,509.81 \times 0.199252 = \$10,263.43$$

We need ensure that comparisons are made over the same time frame, so we calculate the PV of an infinite number of replacement cycles. The AEV for one cycle is the same as for an infinite number of cycles:

$$AEV_{\text{One cycle}} = AEV_{\infty \text{ cycles}} = PV_{\infty \text{ cycles}} \cdot CRF_{0.15}^{\infty} = PV_{\infty \text{ cycles}} \cdot i \text{ and therefore } PV_{\infty \text{ cycles}} = \frac{AEV_{\text{One cycle}}}{i}$$

$$PV_{\text{New } \infty \text{ cycles}} = \frac{10,263.43}{i} = \frac{10,263.43}{0.15} = \$68,422.88$$

Consider the cash flow of immediately replacing the existing HVAC unit with the new unit – The PV of an infinite number of cycles of the new unit less the salvage value of the existing unit.

$$PV_{\text{New immediate replacement}} = \$68,422.88 - \$8,000 = \$60,422.88$$

Consider the cash flow of replacing the existing HVAC unit with the new unit after one year – The PV of an infinite number of cycles of the new unit at the beginning of year 1. There is no salvage value.

$$PV_{\text{New replace after one year}} = \$68,422.88 \times SPPWF_{0.15}^1 = 68,422.88 \times 0.869565 = \$69,498.15$$

Now consider the cash flow of replacing the existing HVAC unit with the new unit after two years – The PV of an infinite number of cycles of the new unit at the beginning of year 2 and a major overhaul of the existing unit after one year. There is no salvage value.

$$\begin{aligned} PV_{\text{New replace after two years}} &= 68,422.88 \times SPPWF_{0.15}^2 + 9,000 \times SPPWF_{0.15}^1 \\ &= 68,422.88 \times 0.869565 + 9,000 \times 0.86956 \\ &= \$59,563.64 \end{aligned}$$

The lowest Present Value Cost (PVC) is the option of replacing the existing HVAC unit with a new model of the unit after one year, but there is little difference in the PVC – Replace immediately (\$60,422.88), replace after one year (\$59,498.15), and replace after two years (\$59,563.64). Other factors of risk (see later section) may influence the final decision.

EXAMPLE 6 – AUGMENTATION VERSUS REPLACEMENT

A building has an existing HVAC unit which is now too small because of an additional heat load due to computer equipment. The combined annual operating and maintenance costs are \$1,300. The unit has a resale value now of \$1,500, a reassessed service life of 6 years with a salvage value then of \$300.

It is possible to satisfy the additional heat load by augmenting the existing HVAC unit with an additional new HVAC unit costing \$3,500, a service life of 8 years, salvage value of \$350, and combined annual operating and maintenance costs of \$1,200.

OR

Immediately replace the existing HVAC with a new larger HVAC unit which would cost \$6,000. The new larger unit has a service life of 10 years, salvage value of \$600, and a combined annual operating and maintenance cost of \$2,000. The building owner's cost of capital is 15% per annum.

What is your recommendation?

First consider the PV of one cycle of the larger unit. The cash flows are the initial capital cost, the annual operating and maintenance costs, and the salvage value:

$$\begin{aligned}
 PV_{\text{Large one cycle}} &= 6,000 + 2,000 \times USPWF_{0.15}^{10} - 600 \times SPPWF_{0.15}^{10} \\
 &= 6,000 + 2,000 \times 5.01877 - 600 \times 0.247185 \\
 &= 600 + 10,037.54 - 148.31 \\
 &= \$15,889.23
 \end{aligned}$$

$$AEV_{\text{Large one cycle}} = 15,889.23 \times CRF_{0.15}^{10} = 15,889.23 \times 0.199252 = \$3,165.96$$

$$PV_{\text{Large } \infty \text{ cycle}} = \frac{3,165.96}{i} = \frac{3,165.96}{0.15} = \$21,106.41$$

Secondly consider the PV of one cycle of the additional smaller unit. The cash flows are the initial capital cost, the annual operating and maintenance costs, and the salvage value:

$$\begin{aligned}
 PV_{\text{Small one cycle}} &= 3,500 + 1,200 \times USPWF_{0.15}^8 - 350 \times SPPWF_{0.15}^8 \\
 &= 3,500 + 1,200 \times 4.48732 - 350 \times 0.326902 \\
 &= 3,500 + 5,384.78 - 114.42 \\
 &= \$8,770.36
 \end{aligned}$$

$$AEV_{\text{Small one cycle}} = 8,770.36 \times CRF_{0.15}^8 = 8,770.36 \times 0.222850 = \$1,954.47$$

$$PV_{\text{Small } \infty \text{ cycle}} = \frac{1,954.47}{i} = \frac{1,954.47}{0.15} = \$13,029.83$$

If we immediately replace the existing unit with a larger unit, then the PV of this option is simply the PV of the larger unit (\$21,106.41) less the immediate resale value (\$1,500) of the existing unit i.e. \$19,606.41

If we go for the augmentation option we need to also estimate the PV of retaining the existing unit. Both the existing unit and the smaller unit should be replaced by a larger unit at the same time.

A home exercise is to estimate the PV of the augmented option when the remaining service life of the existing unit is:

- a) 6 years as above
- b) 8 years which coincides with the service life of the smaller unit.

ADVANTAGES OF NPV

- The NPV technique satisfies the three necessary criteria for investment appraisal – it uses all available data, it takes account of the time value of money, and it generates a return at least equal to the market rate of interest.
- Where the economic life of a project can be fairly well estimated the NPV always indicates the favoured option.

DISADVANTAGES OF NPV

- Because Net Present Value is expressed in terms of one particular year – usually the year of commencement or the year of commissioning – the figure changes if moved to a different base year.
- The resulting information does not make visible the dynamic nature of the process on a year-to-year basis. Alternatives with the same NPV may have different payback periods and hence less risk.

EFFECT OF SCALE

- A common misconception about NPV as a criterion is that it does not capture the effect of scale.
- A project yielding \$101 for a cost of \$100 is as good as a project yielding \$11 for the cost of \$10.
- The scale is included since \$101 includes the net surplus of \$1 over the cost including a normal

return.

ROBUSTNESS

- The scale criticism is in fact a confusion of scale with robustness. Obviously the smaller project is more robust since it could withstand a 10% downside variation while the larger could only withstand a 1% variation.

INTERNAL RATE OF RETURN (IRR)

DESCRIPTION OF IRR

- The Internal Rate of Return (IRR) of a project is the discount rate at which the benefits equate to costs.
- It is the average return on resources employed over the life of a project.

METHOD OF IRR

- For the internal rate of return (IRR), find the value of d such that

$$NPV = \sum_{t=0}^T \frac{B_t - C_t}{(1+d)^t} = 0$$

where $B_t - C_t$ is the Net Return, or Net Cash Flow, received in any year t .

- The technique of IRR does not consider the source of the funds in the present, the opportunity costs of these funds other than the "cost of capital", or discount rate, nor of the use or disposition of the Net Returns, $B_t - C_t$, in future time periods. Excepting the compounding of interest, there is nothing said or implied about reinvestment (Dudley, 1972).
- At the internal discount rate the net present value of a project switches from loss making to profitability.

SELECTION CRITERIA OF IRR

- Rank projects by their IRR.
- Mutually exclusive projects and limited finance: select the project with the highest IRR provided $IRR > i$.
- Independent projects and limited finance: select projects in descending order of IRR until the budget is exhausted. Eliminate those projects where $IRR < i$.

EXAMPLE 1

Is it while worthwhile to receive \$2000 now instead of \$156 per annum for 16 years?

- In equation form

$$2000 = 156 \times USPWF_i^{16}$$

- If $i = 2$ percent,

$USPWF = 13.578$ and RHS of equation = \$2120.

If $i = 3$ percent,

$USPWF = 12.561$ and RHS of equation = \$1960.

By linear interpolation,

$$i = 2 + \frac{2120 - 2000}{2120 - 1960} = 2.75 \text{ percent}$$

- At a discount rate of 2.75% a present sum of \$2000 and a uniform series of \$156 per annum for 16 years are equal in value.
- The current interest rate (real or nominal) is much higher. It would not be worthwhile to invest \$2000 at 2.75% in order to save paying \$156 per annum for 16 years.
- A higher rate of interest means greater discounting a lower present worth for the uniform series therefore it would be worthwhile to receive \$2000 now in preference to \$156 per annum for 16 years.

EXAMPLE 2

A firm is faced with the problem of choosing from two machines A and B, both capable of doing the same work.

	Machine A	Machine B
First Cost	\$20,000	\$14,000
Estimated Life	10 years	10 years
Salvage Value	Nil	Nil
Annual Running Costs	\$2000	\$3000

- As a rule a more expensive alternative will either produce savings or additional income. The problem is to establish whether the extra investment can be justified.
- Machine A costs \$6,000 extra but produces an annual savings in running costs of \$1000. What is the rate of return of the extra investment?
- Equating this extra investment with the present worth of the annual savings

$$6000 = 1000 \times USPWF_i^{10}$$

$i = 10$ percent, $USPWF = 6.144$, $RHS = 6144$

$i = 12$ percent, $USPWF = 5.560$, $RHS = 5650$

Interpolating, $i = 10.3$ percent.

- If the company's minimum rate of return is less than the internal rate of return on the additional investment then Machine A, the more expensive machine, can be justified.

EXAMPLE 3

A firm is faced with the problem of choosing from two machines C and D, both capable of doing the same work. The firm's specified minimum rate of return is 20 percent.

	Machine C	Machine D
First Cost	\$20,000	\$30,000
Annual Operating Cost	\$3000	\$1500
Major Overhaul Cost	\$5000 at 8 years	\$3000 at 10 years
Salvage Value	\$3000	\$5000
Economic Life	15 years	15 years

- At what discount rate does the extra investment equal the present worth of the annual savings?

$$(30,000 - 20,000) + (3000 \times SPPWF_i^{10} - 5000 \times SPPWF_i^8) - (5000 - 3000) \times SPPWF_i^{15}$$

$$= (3000 - 1500) \times USPWF_i^{15}$$

- Simplifying,

$$10,000 = 1,500 \times USPWF_i^{15} - 3,000 \times SPPWF_i^{10} + 5,000 \times SPPWF_i^8 + 2,000 \times SPPWF_i^{15}$$

- Try $i = 12$ percent

$$1500 \times 6.811 - 3000 \times 0.322 + 5000 \times 0.4039 + 2000 \times 0.1827 = \$11,640.$$

- Try $i = 15$ percent

$$1500 \times 5.847 - 3000 \times 0.2472 + 5000 \times 0.3269 + 2000 \times 0.1229 = \$9920.$$

- The extra investment required by D will show a rate of return slightly under 15 percent so will not be recommended as the rate is considerably less than the firm's specified minimum of 20 percent.
- Refer to diagram showing the effect of the Rate of Return on Economic Decision-making.
- When the minimum rate of return is greater than 15 percent, the extra investment of \$10,000 exceeds the present worth of annual savings and should be rejected. When it is less than 15 percent, the present worth of annual savings exceeds the extra investment and therefore justifies it.

ADVANTAGES OF IRR

- It is useful for looking back at post-mortems of projects since the figure is not affected by base dates.
- People find it easier to understand a rate of return concept because of familiarity with interest rates.
- Unlike the Annual Equivalent Value (AEV) and Net Present Value (NPV) methods, the IRR technique does not require a predetermined interest rate or rate of return. Rather, it determines what this rate is and uses it to assist decision-making.

DISADVANTAGES OF IRR

- It is difficult to calculate without the aid of a financial calculator or computer.
- In certain circumstances the technique will give more than one IRR. This occurs when a project has relatively large net costs at more than one stage of its life.
- IRR has an inherent bias in favour of projects with a faster payback period compared to projects which generate slower returns but which will provide the same, or greater, Net Present Value at the minimum acceptable rate of return. (Refer to AIRR and reinvestment of Net Returns in next section).

LIMITATIONS OF IRR

In choosing how large an investment to make the IRR is appropriate only if it is calculated for marginal investment changes. Mistakenly using IRRs based on total costs and benefits (savings) of each size will result in uneconomical size choices.

ADJUSTED INTERNAL RATE OF RETURN (AIRR)

DESCRIPTION OF AIRR

- The Adjusted Internal Rate of Return measures over the life of a project the combined return on the original investment and on the reinvested earnings, allowing for the reinvestment rate(s) to differ from the rate earned on the original investment.

BACKGROUND OF AIRR (Dudley, 1972)

- The ultimate criterion between two alternative courses of action is the total wealth that an investor can expect from each alternative by the terminal date of the longer-lived project. In order to make a fair comparison, an explicit assumption must be made regarding the rate at which funds released by either project can be reinvested up to the terminal date.
- Any assumptions made about the reinvestment rate of return up to a common horizon point are not implicit in the technique of determining either the present value or the internal rate of return.
- Any assumptions about reinvestment are implicit, unknowingly and by default, by not making any explicit estimate of the possible return on reinvestment of intermediate cash flows prior to the terminal date of the longer-lived project.

METHOD OF AIRR (Marshall, 1986)

- The solution value of i in the following equation is the AIRR

$$\sum_{t=1}^N \frac{(B_t - C_t)(1 + r_t)^{N-t}}{(1 + i)^N} - C_0 = 0$$

where r_t = the rate of return on reinvestment of net cash flows realised in period t and compounded at that rate for $N - t$ periods.

- Net cash flows ($B_t - C_t$) are carried forward (compounded) at the appropriate reinvestment rates (r_t) to equivalent values at the end of the study period (end of the N th period) and summed.
- The value of i that discounts the resulting future value to a present value equal to C_0 (initial project cost as of the base time) is the AIRR. The higher is the assumed reinvestment rate, the higher will be the calculated value of the AIRR.
- If the assumed reinvestment rate is the same for all cash flows and equal to the unadjusted IRR (i.e., when $r_t = i$) the AIRR and IRR technique will yield the same solution value (i.e., AIRR = IRR).
- The more concentrated the net cash receipts are early in the study period, and the greater the difference between reinvestment rates and the IRR, the larger will be the difference between the AIRR and the IRR for any given investment. It will be more important to use the AIRR to find the true return on an investment.
- If r_t changes from one period to the next, and if some values are larger and others smaller than i , then the AIRR and IRR measures would have to be computed for each project to determine which measure was larger.
- The AIRR can be solved for directly by rearranging the above equation.

$$AIRR = \left(\frac{TV}{C_0} \right)^{\frac{1}{N}} - 1$$

where

$$TV = \sum_{t=1}^N (B_t - C_t)(1 + r_t)^{N-t}$$

i.e., the terminal (future) value (TV) at the end of the study period of all net cash flows over the life of the investment.

- The terminal value (TV) must first be calculated by applying the single compound amount factor (SCAF), computed as $(1 + r_t)^{N-t}$, to each net cash amount and then summing them.

SELECTION CRITERIA

- Rank projects by their AIRR.
- Mutually exclusive projects and limited finance: select the project with the highest AIRR provided $AIRR > i$.
- Independent projects and limited finance: select projects in descending order of AIRR until the budget is exhausted. Eliminate those projects where $AIRR < i$.

EXAMPLE 1

A project has an initial cost of \$2,200 and yields a positive net cash flow of \$1000, \$1500, and \$1000 over a period of three years. The first and second year's net cash flow are reinvested at 20 percent and 15 percent respectively. What is the AIRR of the project?

Year (t)	Net Cash Flow ($B_t - C_t$)	Reinvest Rate (r_t)	N-t	SCA Factor ($1 + r_t$) ^{N-t}	Terminal Value
0	\$-2,200				
1	+1,000	20%	2	1.440	\$1,440
2	+1,500	15%	1	1.150	1,725
3	+1,000	0%	0	1.000	1,000
TOTAL	\$1,300				\$4,165

$$AIRR = \left(\frac{4,165}{2,200} \right)^{\frac{1}{3}} - 1 = 0.237 \text{ or } 23.7\%$$

ADVANTAGES OF AIRR

- The AIRR approach is a more accurate measure than IRR because it incorporates the expected real earnings of reinvested net cash flows.
- The AIRR is easier to compute than the IRR because a direct solution of the AIRR is possible whereas iterative procedures such as the trial-and-error approach are required to arrive at an approximation for the IRR. Computers make the computational differences less important.
- The AIRR approach results in one unique solution thus avoiding a major limitation of the IRR where there may be multiple solutions.

LIMITATIONS OF AIRR

- As with IRR, in choosing how large an investment to make the AIRR is appropriate only if it is calculated for marginal investment changes. Mistakenly using AIRRs based on total costs and benefits (savings) of each size will result in uneconomical size choices.
- AIRR rankings will not indicate the optimal project mix (maximisation of mix of Net Present Values) in every case when choosing among different purpose projects competing for the same budget.

BENEFIT COST RATIOS

DESCRIPTION OF BENEFIT COST RATIOS

- The benefit-cost ratio is a measure of the value or benefit, expressed in dollar amounts, achieved from a dollar spent. The costs used in the denominator are the increases in the costs of implementing, operating, and maintaining an alternative over the base condition while the benefits (numerator) are the differences in costs incurred and benefits gained as a result of the expenditure.

METHOD OF BENEFIT COST RATIOS

$$B/C = \frac{\sum \text{Present value of benefits}}{\sum \text{Present value of costs}}$$

ACCEPTANCE CRITERIA OF BENEFIT COST RATIOS

- Acceptance criterion requires that the B/C ratio be greater than or equal to 1.0. If the ratio is exactly 1.0 the project is called marginal.

ADVANTAGES OF BENEFIT COST RATIOS

- This method of analysis is particularly effective when the time period or initial capital investment differences amongst alternatives are significant.
- It is also useful in the selection of a preferred investment when the NPVs are nearly equal, because the higher benefit/cost ratio indicates a greater rate of return, and a quicker payback combined with reduced risk.

DISADVANTAGES OF BENEFIT COST RATIOS

- Benefit-cost ratios will give misleading results if cost savings are shown as benefits or reduced benefits as costs.

INCREMENTAL BENEFIT-COST RATIO

DESCRIPTION OF INCREMENTAL BENEFIT COST RATIO

- Procedure used to determine whether each additional investment produces an additional gain greater than its cost.

METHOD OF INCREMENTAL BENEFIT COST RATIO

$$\text{Incremental } B/C \text{ ratio} = \frac{PV \text{ of benefits}}{PV \text{ of costs}}$$

- Array alternatives in ascending order of cost.
- Starting at the lowest cost alternative, consider the next-higher-cost alternative and compute the ratio of the present value of incremental benefits to the present value of incremental costs, where incremental benefits and costs are those in excess of the benefits and costs of the lower cost alternative.

SELECTION CRITERION OF INCREMENTAL BENEFIT COST RATIO

- Ratios greater than 1.0 implies that the incremental benefits are greater than and therefore justify the incremental cost. Thus if the ratio is greater than or equal to 1.0 discard the lower cost alternative and make the higher cost alternative the basis for comparison with the next-higher-cost alternative.
- If the incremental benefit-cost ratio is less than 1.0 the higher cost alternative is discarded and the lower cost alternative used as the basis for the next increment.
- Repeat procedure until all alternatives have been analysed.
- Select the alternative with the highest capital cost whose incremental benefit-cost ratio is equal to or greater than 1.0.

BREAK-EVEN ANALYSIS

DESCRIPTION OF BREAK-EVEN ANALYSIS

- Break-even analysis is the determination of the conditions under which an investor can recuperate an investment when one of the factors is unknown and must be determined – initial investment, minimum life (payback period), or annual benefits or savings.
- Benefits and costs can be discounted or, more typically, are not discounted.
- When benefits and costs are discounted the investment is recovered when the NPV curve changes from negative to positive (NPV = 0). Equating the NPV to '0' and inputting the two known values to determine the third and unknown value may therefore solve the value of the unknown factor.

EXAMPLE OF BREAK-EVEN ANALYSIS

The fixed costs of a large-scale house builder amounted to \$100,000 a year. The variable cost (in 1969) is estimated to be \$4000 per house. He expects to be able to sell his houses at \$6000 each. What is the break-even point?

- Let x be the number of houses built and sold per year.

$$100,000 + 4000x = 6000x, \text{ or } x = 50.$$

- At 50 houses, the builder is breaking even and is able to cover his costs. His profit does not start until he sells his 51st house. Reducing the sale price without reducing costs would automatically push the break-even point higher. Say the sale price is reduced by \$400,

$$100,000 + 4,000x = 5,600x, \text{ or } x = 63.$$

- At the reduced price, the builder has to sell more to reach the break-even point. A lower margin requires higher volume to maintain profitability. Increasing the sales price has the opposite effect by reducing the break-even point.
- Refer to diagram

ADVANTAGES OF BREAK-EVEN ANALYSIS

- The technique is useful for marketing and assessment of exposure to risk.

DISADVANTAGES OF BREAK-EVEN ANALYSIS

- The method does not take into account the expected life of the project or flows occurring after the test point.
- The technique is not a guide to the profitability of a project.

PAYBACK

DESCRIPTION OF PAYBACK

- The payback period is the time required for revenues to balance costs.
- Revenues and costs can be discounted or more typically are not discounted.

METHOD OF SIMPLE PAYBACK

$$\text{Simple payback period} = \frac{\text{Initial Investment}}{\text{annual cost saving}}$$

DISCOUNTED PAYBACK PERIOD

- When a running cumulative NPV is calculated, the year in which the NPV changes from negative to positive is the discounted payback period. This is the point at which the initial investment is re-paid in full from future revenues generated, including the interest on the investment during this period.

ADVANTAGES DISCOUNTED PAYBACK PERIOD

- It is useful to use as an initial screening device before the application of more powerful methods of appraisal.
- The simple payback period is quick and simple to calculate.
- The discounted technique is useful for marketing and assessment of exposure to risk. The least risk situation is identified by the shortest payback period before the expiry of anticipated physical life.

DISADVANTAGES DISCOUNTED PAYBACK PERIOD

- The method does not take into account the expected life of the project or flows occurring after the test point.
- The technique is not a guide to the profitability of a project.

UNCERTAINTY AND RISK

UNCERTAINTY

- Uncertainty (or certainty) refers to state of knowledge about the variable inputs to an economic analysis. If the analyst is unsure of input values, there is uncertainty.
- Sensitivity analysis is an example of a technique that accounts for uncertainty. It provides a range of measures of project worth that correspond to a range of possible input values.

RISK EXPOSURE

- Risk exposure is the probability of investing in a project that will have a less favourable than what is desired (the target) or from what is expected.
- Probability distributions of measures of project worth describe risk exposure. Simulation is an example of a technique that can be used to build probability distributions that show risk exposure.

RISK ATTITUDE

- Risk attitude, also called risk preference, is the willingness of a decision-maker (or investor) to take a chance or gamble on an investment of uncertain outcome.
- A given investment of known risk might be economically acceptable to an investor who is a risk taker, but totally unacceptable to another investor who is risk averse.
- Decision analysis is an example of a technique that uses utility functions to account for risk attitude.

RISK EXPOSURE

- A probability distribution quantifies risk exposure by showing probabilities of achieving different economic worth values.
- Refer to diagram of risk exposure.
- The smaller the spread of the distribution, as measured by the standard deviation, the tighter the distribution around the mean value and the smaller is the risk associated with the project.
- Although project D has the larger mean, it also has the greater variance. That is, the project with the greater expected return also has the greater variance or risk of exposure.
- Some procedure for including risk attitude in project evaluation is required to establish project preference in this case.

RISK ATTITUDE

- Risk attitude is measured by the trade-offs decision-makers will make between uncertain money payoffs of known probability and sure money payoffs.
- These trade-offs are determined by asking decision makers to specify how much more money (the certainty equivalent) must be received to make them indifferent between the certainty equivalent and the expected value of a given amount that is not certain.
- For example, if a person were given a choice of a 50% probability of winning \$1000 and a sure certain amount of money there would be some trade-off at which the person would be indifferent between the lottery and the sure payment.
- The revealed trade-offs between the expected value of the lottery and the sure payment determine whether a decision-maker is risk neutral, risk averse, or a risk taker.
- Once several such trade-offs have been specified, a relationship between money and utility can be determined.
- Utility here means simply the satisfaction a decision-maker receives from given quantities of money.
- Refer to diagram of utility function.
- The utility function reflects a particular relationship between satisfaction, a subjective value, and monetary amounts.
- The utility function is unique to one individual, firm, or institution.

RISK AVERSE

- If the utility function bends over to the right, the decision maker is risk averse. Increasingly large dollar amounts are required to achieve constant increments of utility; the marginal utility of income is decreasing.
- The decision-maker would prefer a sure payment that is less than the expected value of the lottery to the chance of participating in the lottery.
- The decision-makers regard marginal payoffs to be worth less (of less utility) per dollar, as total payoffs increase.
- To determine the true value of investments for risk-averse decision-makers, economic analyses must account for decreasing satisfaction of higher payoffs with corresponding decreases in marginal utility.

RISK TAKER

- If the utility function bends upward to the left, the decision-maker is a risk taker. Successively smaller dollar amounts are required to achieve constant increments in utility; the marginal utility of income is increasing. The decision-maker regards project payoffs to be worth more (more utility) per dollar, as the total payoffs increase.

MULTI-RISK ATTITUDE

- Behaviour of decision-makers implies that more than one risk attitude is possible, depending on the monetary stakes.
- For example, many low-income persons are willing to buy insurance at a premium greater than the expected value of a loss without insurance (sign of risk aversion) and at the same time to play the lottery at worse than fair odds (sign of risk taker).
- Refer to diagram of risk averse and risk taking attitudes.