Environment and Planning A 1998, volume 30, pages 1295 – 1311

The optimum timing and maximum impact of full rehabilitation of New Zealand housing stock

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Received 20 January 1996; in revised form 11 April 1997

Abstract. The author develops a simulation model to estimate the optimum timing and maximum impact of full rehabilitation of New Zealand housing stock. The model is based on the theories of classical population dynamics. Data used in the model include empirical estimates of the mortality of New Zealand housing stock, assumed schedules of depreciation of dwelling services, and assumed schedules of annual maintenance costs. The dwelling service years provided by dwellings serve as a proxy for benefits of rents or imputed rents (excluding rent for land). The cost to construct one dwelling and fractions thereof serve as a proxy for costs of maintenance, rehabilitation, replacement, and new construction. Optimum timing of rehabilitation can increase the quantity of benefits provided by the housing stock per unit total cost but a reduction in the growth rate of new dwellings has a greater impact in achieving the same objective. A stationary and stable housing stock can provide 45% more dwelling services per unit total cost than a housing stock which doubles in size every 35 years.

1 Introduction

The economic maxim that the use of resources should be optimised applies to the considerable resources embodied in existing housing stock. In a recent perpetual inventory study of real capital in New Zealand, Philpott (1992) estimated that dwellings in 1989 formed over 23% of the total value of the nation's capital stock of buildings, infrastructure, plant, and equipment. Government and local authorities tend to provide public housing by investing in new construction and disregard or overlook the option of investing in the rehabilitation of existing dwellings. Although investment in new construction provides immediate housing which satisfies an immediate need, investment in rehabilitation also provides additional housing in the long run by extending the life expectancy of dwellings. Gleeson (1992) and Johnstone (1995; 1997) have developed actuarial models to estimate when investment is better diverted from new housing to rehabilitation of existing housing stock for each age at which rehabilitation takes place. These actuarial models in isolation do not take into account the optimum timing of rehabilitation. By extending the life expectancy of dwellings, rehabilitation also reduces the replacement rate of the housing stock. The level of reduction in the replacement rate is determined by the timing of rehabilitation. For example, if only older dwellings undergo rehabilitation, then the life expectancy of a smaller number of dwellings is extended and reductions in the replacement rate are smaller as a result. The actuarial models also do not take into account differences in the quality of dwelling services provided by a housing stock when undergoing different rates of expansion. The mean age of an expanding housing stock is less than that of a stationary housing stock and, ceteris paribus, younger dwellings undergo less depreciation.

In this paper I develop a simulation model to estimate the optimum timing and maximum impact of rehabilitation of New Zealand housing stock. The interacting dynamics between benefits in the form of dwelling services and costs, including that of maintenance, replacement construction, and new-build construction which adds to the size of a housing stock are modelled in the process. In order to estimate the maximum impact of rehabilitation, dwelling are assumed to undergo 'full' and extensive rehabilitation as defined later in this paper. Partial rehabilitation is not examined.

2 Description of simulation model

The simulation model is a dynamic housing stock model based on the theories of classical population dynamics (Keyfitz, 1968) and is structured as an interlinking series of survivorship schedules of dwelling cohorts that enter the housing stock over successive time intervals. Survivorship is determined by a probability of loss or mortality schedule that forms the first column of a standard life table (see appendix A). The model simulates dwelling losses from each dwelling cohort, and the sum total of these losses over each time interval forms total dwelling losses of all ages. These total dwelling losses are replaced by replacement construction. New-build construction adds to the size of the housing stock in contrast to replacement construction which replaces total dwelling losses. The housing stock undergoes expansion when new-build construction is greater than zero. Dwelling cohorts that are still standing at the start of each time interval form the state variables of the simulation model.

2.1 Homogeneity and real constant costs

The size and quality of dwellings within the simulation model are assumed to be homogeneous for the sake of model simplicity. Heterogeneity can be taken into account by constructing separate simulation models for distinct classes of dwellings and then by combining the results of each model.

The real costs of maintenance, rehabilitation, replacement construction, and new-build construction are assumed to remain constant for the sake of model simplicity. It follows that the long-run supply curve of the construction industry is perfectly elastic and returns to scale are constant as the construction industry expands.

2.2 Mortality of housing stock

The simulation model is driven by schedules of probability of loss data based on my (Johnstone, 1994) empirical study of the mortality of New Zealand housing stock. Gleeson (1985) estimated the mortality of a sample of Indianapolis dwellings in a pioneering study and, more recently, Komatsu et al (1994) estimated the mortality of Japanese timber dwellings. These studies make use of cross-sectional data that require the adoption of the implicit assumption that all dwelling cohorts have been, and will continue to be, exposed to the same regime of mortality. In other words, mortality is assumed to be static.

I previously established (Johnstone, 1994) that the mortality of New Zealand housing stock is a function not only of age, but also of the annual expansion rate of the stock by using a simulation model similar to that developed in this paper. Between 1860 to 1990 the probability of loss of dwellings had increased and decreased with each increase and decrease in the annual expansion rate. The average life expectancy of dwellings upon entry to a housing stock increases when the probability of loss decreases. For example, the current average life expectancy of New Zealand housing stock would increase from 90 years to 130 years should the annual expansion rate of the housing stock decline to zero.

2.3 Expenditure on full rehabilitation

Dwellings are assumed to undergo 'full' rehabilitation. The effective age of dwellings that undergo full rehabilitation is zero immediately after undergoing rehabilitation. These dwellings subsequently provide the same level of dwelling services (excluding land) as that of new dwellings and these dwellings undergo the same schedule of depreciation. Expenditure on full rehabilitation is assumed to be no greater than that which can be justified on an actuarial benefit to cost ratio basis. Elsewhere (Johnstone, 1995; 1997) I have developed an actuarial model to estimate the maximum expenditure that can be justified rehabilitating New Zealand dwellings. Estimates are made for each age at which full rehabilitation takes place. The maximum expenditure is expressed as a ratio of the costs to construct a new dwelling of similar size and quality; hence the 'maximum cost ratio' name of the model.

Greater maximum cost ratios are justified for all ages at which full rehabilitation takes place when benefits and costs of full rehabilitation and new construction are discounted by using low discount rates (Johnstone, 1995; 1997). Low discount rates therefore favour full rehabilitation.

The maximum cost ratio may exceed unity under schedules of increasing maintenance costs with age. An expenditure on full rehabilitation greater than the cost of a new dwelling would then be justified. Early demolition and replacement would also be justified. This latter option is assumed to be the favoured option.

2.4 Full rehabilitation of entire dwelling cohorts

The existence of maximum cost ratios is no guarantee that dwellings can undergo full rehabilitation within the maximum justifiable budget. A proportion of each dwelling cohort cannot, and should not, undergo full rehabilitation. Nonetheless, entire national dwelling cohorts are assumed to undergo full rehabilitation within budget. This extreme assumption enables estimates of the maximum impact of rehabilitation. Dwelling cohorts are also assumed to undergo full rehabilitation once only for the sake of simplicity of model construction. These assumptions are illustrated in figure 1 which shows the surviving stock schedule of a national dwelling cohort where an entire surviving cohort undergoes full rehabilitation at the age of 75 years. The surviving stock schedule initially follows the same course as that for a new dwelling cohort. In the process, a number of dwellings are lost over the time interval between entry to the housing stock and the age at which full rehabilitation takes place. Upon full rehabilitation at the age of 75 years the remaining schedule then follows a similar, but scaled down, course. The end result is that the economic life span of the original dwelling cohort is extended from 130 years to 210 years. The life expectancy upon entry of the original dwelling cohort is also extended.

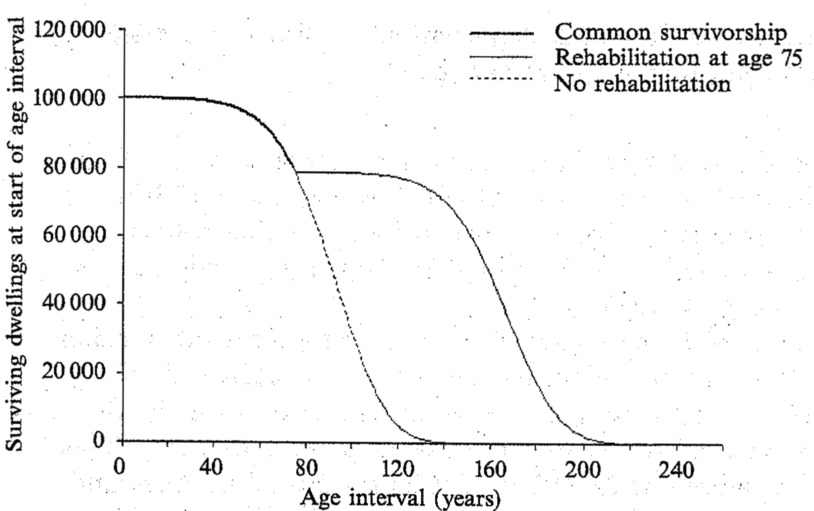

Figure 1. Survivorship schedule of dwellings undergoing full rehabilitation at the age of 75 years.

2.5 Depreciation of dwelling services

All dwellings are assumed to enter the housing stock at the effective age of zero. Dwelling entries include not only new construction but also conversions from existing commercial buildings to residential units and conversions from existing single-unit dwellings to multiunit dwellings. The effective age of these dwellings is zero, or close to zero, after undergoing extensive retrofitting.

Depreciation of dwelling services subsequent to entry is described by the function $D(h)$, where h is the effective age of a dwelling cohort. The depreciation factor $D(h)$ is expressed as the proportion of dwelling services provided by a new dwelling cohort in its first year of life ($0 \leqslant D(h) \leqslant 1$). If a dwelling cohort has yet to undergo rehabilitation or does not undergo rehabilitation, then the effective age of the dwelling cohort is given by $h = x$, where x is the actual age of the dwelling cohort at the start of the age interval x to $(x+1)$. The symbol x is used as a variable in this context. If an entire dwelling cohort undergoes full rehabilitation, then the effective age of the dwelling cohort is $h = 0$ immediately following rehabilitation. The subsequent effective age is $h = x - y$, where y is the age at which the dwelling cohort underwent full rehabilitation.

An empirical study of the depreciation of New Zealand housing stock has yet to be carried out. Extensive literature surveys of empirical studies of depreciation of dwellings by Malpezzi et al (1987) and Baer (1991) do not provide satisfactory guidelines which can be applied with confidence to New Zealand housing stock. No study estimates the depreciation of dwelling services or rent (excluding rent for land) over the full economic life span of dwellings. Two schedules of depreciation of dwelling services are therefore assumed in this paper.

Depreciation schedule 1 is assumed to be straight-line described by the equation

$$D(h) = 1 - j(h+1), \tag{1}$$

where $j = 0.8\%$ per year. Under straight-line depreciation, dwelling services depreciate to a value of zero by the age of 125 years. This age approximates the current 130-year economic life span of New Zealand housing stock.

Depreciation schedule 2 is assumed to be diminishing value depreciation described by the equation

$$D(h) = \left(\frac{1}{1+j}\right)^{h+1}, \tag{2}$$

where $j = 1.0\%$ per year. Under diminishing value depreciation dwelling services decline to 27% of the original value at entry by the age of 130 years.

2.6 Annual maintenance costs

The annual maintenance costs of a dwelling cohort over an age interval x to $(x+1)$ are estimated as the product of the maintenance factor described by the function $M(h)$ and L_x which is the number of dwelling service years provided by a dwelling cohort over that age interval. The maintenance factor $M(h)$ is expressed as a proportion of the costs to construct a new dwelling ($0 \leqslant M(h) \leqslant 1$).

The best available data on annual maintenance costs of New Zealand housing stock consist of records of New Zealand Housing Corporation (NZHC) dwellings that date back to the early 1940s. I estimate the average annual costs to maintain a New Zealand dwelling to be 1.0% of the costs to construct a new dwelling. This estimate is based on a random sample of 25 NZHC dwellings located in Auckland. As records of maintenance fall well short of the 130-year economic life span of New Zealand housing stock, in this paper I therefore assume three different schedules of maintenance costs based on the above estimate.

Under maintenance schedule 1 annual maintenance costs are assumed to decline exponentially from 1.0% of the costs to construct a new dwelling at the age of zero to 0.5% by the age of 130 years. Under maintenance schedule 2 annual maintenance costs are assumed to remain constant at 1.0% of the costs to construct a new dwelling over the full economic life span of housing stock. Under maintenance schedule 3 annual maintenance costs are assumed to increase exponentially from 1.0% of the costs to construct a new dwelling at the age of zero to 2.0% by the age of 130 years. The effectiveness of maintenance is assumed to be the same under all maintenance schedules.

2.7 Benefit to cost ratio criterion and proxies for benefits and costs

In order to optimise the use of resources required to sustain dwelling services, alternative investment streams allocated to maintenance, rehabilitation, replacement construction, and new-build construction need to be increased or decreased in any proportion. A ratio of present-value benefits to costs criterion is therefore used instead of the excess benefit over cost criterion to rank alternative investment streams (Mishan, 1982).

The excess benefits to costs ratio and the benefits to costs ratio criteria both give the same rankings of alternative investment streams. The excess benefits to costs ratio criterion requires that benefits and costs be measured in the same metric, otherwise like is deducted from unlike. In contrast, the benefit to cost ratio criterion does not require benefits and costs to be in the same metric in order to correctly rank alternative investment streams. This is demonstrated in appendix B.

Benefits and costs in this paper are expressed in units of quantities, instead of value, in order to simplify the model. Dwelling service year equivalents (sye) serve as proxy for benefits. This proxy takes into account the decline in quality of dwelling services because of depreciation. Dwelling construction units (cu), the costs to construct one dwelling, serve as a proxy for costs.

A disadvantage of using a quantity of benefits to quantity of costs ratio criterion to rank alternative investments is that this ratio does not indicate whether the value of benefits exceeds or matches the value of costs. The quantity of benefits to quantity of costs ratio are therefore converted to a value of benefits to value of costs ratio as a check to ensure the simulation model produces sensible and realistic results.

2.8 The objective function

When a housing stock is stationary and stable the distribution of dwelling cohorts by age is the same over each successive time interval. The sum total of dwelling service year equivalents provided by the stationary housing stock over each successive time interval therefore remains constant. The total costs required to sustain the stationary housing stock over each successive time interval also remain constant. Benefits due to capital expenditure over each time interval are fully realised over future time intervals and the major proportion of benefits over each time interval is the result of capital expenditure over previous time intervals. Because dwelling services over each time interval are sustained by an expenditure over the same time interval, the benefits provided over each time interval can be regarded as being the direct result of the expenditure over that same time interval. If we do this, current and future benefits and costs only need to be taken into account when we estimate the discounted benefit to cost ratio.

When a housing stock undergoes sustained growth at a constant expansion rate the distribution of dwelling cohorts by age stabilises after a sufficient period of time has elapsed. The time period required to reach stability is dependent on the original distribution by age and it will take at least as long as one economic life span to stabilise fully. The ratio of the size of any dwelling cohort to the size of the total housing stock remains constant when an expanding housing stock has reached a stable state. Benefits and costs of an expanding and stable housing stock therefore increase by the same proportion over each time interval and, for the same reasons as given above, future benefits and costs only need to be taken into account when one is estimating the discounted benefit to cost ratio.

The objective function is simplified when the distribution of dwelling cohorts by age is stable because under these conditions benefits and costs increase by the same proportion over each time interval. The objective function, f, takes the form of the ratio of the sum of the total discounted benefits provided by a stable housing stock over each time interval to the sum of the total discounted costs required to sustain those benefits. Future benefits and costs undergo exponential growth, including zero growth.

$$\mathrm{f}(i, n, r, y, \mu_y) = \frac{{}_nB_t \sum_{t=0}^{\infty} v^t \exp(nrt)}{{}_nC_t \sum_{t=0}^{\infty} v^t \exp(nrt)} = \frac{{}_nB_t}{{}_nC_t}, \tag{3}$$

where

i is the real discount rate,

n is the time interval over which benefits are realised and costs are expended,

r is the annual expansion rate of the housing stock,

y is the age at which full rehabilitation takes place,

μ_y is the maximum cost ratio at the age y given as the proportion of the costs to construct a new dwelling,

${}_nB_t$ is the benefits over the time interval t to $(t+n)$,

${}_nC_t$ is the costs over the same time interval t to $(t+n)$,

v is the real discount factor given by $v = 1/(1+i)$.

The objective function f simplifies to the ratio of benefits over a single time interval to costs over the same time interval.

2.9 Benefits in the numerator of the objective function

Benefits in the numerator of the objective function are expressed as follows:

$$B_t = \sum_{x=0}^{w} L_x \mathrm{D}(h), \tag{4}$$

$$\mathrm{D}(h) = \begin{cases} \mathrm{D}(x), & \text{for } x < y, \quad \text{before rehabilitation}, \\ \mathrm{D}(x-y), & \text{for } x \geqslant y, \quad \text{after rehabilitation}, \end{cases}$$

where

B_t is the benefits over the time interval t to $(t+1)$ (by convention, the subscript for the time interval n is omitted when $n = 1$),

L_x is the number of dwelling service years provided by a dwelling cohort within an expanding or stationary and stable housing stock over the age interval x to $(x+1)$,

$\mathrm{D}(h)$ is a depreciation factor that is a function of the effective age (h) of dwelling cohorts,

x is the actual age of dwelling cohorts at the start of the age interval x to $(x+1)$;

w is the economic life span of the housing stock.

The economic life span (w) of the housing stock is defined here as that age beyond which less than 0.1% of an original dwelling cohort survives. This definition based on Shryock and Siegel (1973) enables sensible comparisons of the economic life span of different housing stocks.

The quality of the housing stock is taken into account because the effects of depreciation of dwelling services are taken into account. The units of benefits in equation (4) are therefore in dwelling service year equivalents.

The benefits L_x provided by each dwelling cohort over the time interval t to $(t+1)$ is given by

$$L_x = l_x - d_x a_0 , \tag{5}$$

where

l_x is the number of dwellings from an original dwelling cohort l_0 which survive to the actual age x,

d_x is the number of dwelling losses from a dwelling cohort of actual age x over the age interval x to $(x+1)$,

a_0 is the average number of dwelling service years provided by dwellings lost over the age interval. The value of $a_0 = \frac{1}{2}$ when $n = 1$ gives sufficiently precise results for the purposes of this paper.

Stock losses (d_x) over each age interval are given by the product of the surviving dwellings at the start of each age interval and the best-fit probability of loss function for New Zealand housing stock as follows (Johnstone, 1994):

$$d_x = l_x q_h (1 + 78.62r)^{0.70} \tag{6}$$

$$q_h = \begin{cases} q_x , & \text{for } x < y, \quad \text{before rehabilitation}, \\ q_{x-y} , & \text{for } x \geqslant y, \quad \text{after rehabilitation}, \end{cases}$$

where

q_h is the probability of loss or the proportion of dwellings that are still standing at the beginning of an effective age interval h to $(h+1)$ which will be lost before reaching the end of the age interval.

Total dwelling losses of all ages which are lost over each time interval t to $(t+1)$ are replaced by replacement construction. If the housing stock undergoes expansion, then new dwelling entries include not only replacement construction but also new-build construction which adds to the size of the housing stock. Dwelling entries over the time interval t to $(t+1)$ are given by

$$l_0 = \sum_{x=0}^{w} d_x + P_{t+1} - P_t = \sum_{x=0}^{w} d_x + (\exp r - 1)P_t = \sum_{x=0}^{w} d_x + (\exp r - 1)\sum_{x=0}^{w} l_x , \tag{7}$$

where

l_0 is the number of new dwelling entries of actual ages zero,

P_t is the size of the housing stock at the start of the time interval t to $(t+1)$,

P_{t+1} is the size of the housing stock at the start of the time interval $(t+1)$ to $(t+2)$.

The size of the housing stock at time t (P_t) is simply the sum of all the dwelling cohorts which are standing at time t. The proportion of dwelling entries (l_0) which are lost during the time interval of entry is negligible and is therefore ignored.

2.10 Costs in the denominator of the objective function

Costs in the denominator of the objective function are made up as follows:

$$C_t = C_{\text{new}} + C_{\text{replace}} + C_{\text{rehab}} + C_{\text{maint}} , \tag{8}$$

where

C_t is the total cost required to sustain dwelling services over the time interval t to $(t+1)$,

C_{new} is the cost of new-build construction that adds to the size of the housing stock,

C_{replace} is the cost of replacement construction,

C_{rehab} is the cost of full rehabilitation,

C_{maint} is the cost of maintenance.

The cost of new-build construction is given by the number of new-build entries to the housing stock from equation (7):

$$C_{\text{new}} = (\exp r - 1) \sum_{x=0}^{w} l_x \,, \tag{9}$$

The cost of replacement construction is given by the number of replacement entries to the housing stock from equation (7):

$$C_{\text{replace}} = \sum_{x=0}^{w} d_x \,. \tag{10}$$

The cost of full rehabilitation is given by

$$C_{\text{rehab}} = \mu_y l_y \,, \tag{11}$$

where l_y is the number of dwellings within a dwelling cohort that undergo full rehabilitation at the age y. The maximum cost ratios μ_y used in the simulation model have been estimated in Johnstone (1997).

The cost of annual maintenance is given by the product of the maintenance factor $\mathrm{M}(h)$ and L_x, the number of dwelling service years provided by dwellings over the age interval x to $(x+1)$.

$$C_{\text{maint}} = \sum_{x=0}^{w} L_x \mathrm{M}(h) \,, \tag{12}$$

$$\mathrm{M}(h) = \begin{cases} M(x) & \text{for } x < y, \quad \text{before rehabilitation}, \\ M(x-y) & \text{for } x \geqslant y, \quad \text{after rehabilitation}. \end{cases}$$

The resulting benefit to cost ratio of the housing stock is simply the average life expectancy of dwellings upon entry (e_0) if depreciation of dwelling services is zero, maintenance costs are zero, the housing stock is stationary and stable, and no rehabilitation takes place.

$$\frac{B_t}{C_t} = \frac{\displaystyle\sum_{x=0}^{w} L_x \mathrm{D}(h)}{l_0 + \mu_y l_y + \displaystyle\sum_{x=0}^{w} L_x \mathrm{M}(h)} = \frac{1}{l_0} \sum_{x=0}^{w} L_x = e_0 \,, \tag{13}$$

where l_0 is replacement construction only. If the New Zealand housing stock were stationary and stable then the above benefit to cost ratio would be 130 dwelling service year equivalents per construction unit and the average life expectancy upon entry of the housing stock would be 130 years.

2.11 Variable constraints on objective function

The annual expansion rate r is constrained as follows:

$$0 \leqslant r \leqslant 0.02 \,. \tag{14}$$

The lower limit constraint represents a stationary and stable housing stock. The upper limit constraint represents a doubling in the size of the housing stock every 35 years.

The age at which rehabilitation takes place (y) is constrained as follows:

$$0 \leqslant y \leqslant 130 \,. \tag{15}$$

The upper level constraint corresponds to the current economic life span of New Zealand housing stock.

The maximum costs of full rehabilitation (C_{rehab}) at each age y are constrained by the maximum cost ratio μ_y estimated in Johnstone (1997) which, in turn, is constrained by the real discount rate (i). The minimum costs of full rehabilitation at each age y are constrained by being greater than or equal to zero.

$$0 \leqslant C_{\text{rehab}} \leqslant \mu_y l_y, \qquad \text{where } 0.06 \leqslant i \leqslant 0.12 . \tag{16}$$

All other constraints are fixed constraints which have already been addressed.

3 Procedure

Combinations of the two assumed schedules of depreciation and the three assumed schedules of annual maintenance form six scenarios. Benefits are automatically maximised because entire dwelling cohorts are assumed to undergo full rehabilitation under each scenario. Benefit to cost ratios are maximised when benefits are maximised and total costs are minimised. All costs other than rehabilitation are fixed under each scenario. Benefit to cost ratios under each scenario are therefore optimised when full rehabilitation takes place at no cost. The maximum possible impact of rehabilitation can be estimated by adopting the assumption that full rehabilitation takes place at no cost.

Full rehabilitation is assumed to be able to take place within a maximum budget. Given this assumption, if dwellings require an expenditure of the full budget in order to undergo full rehabilitation, then a lower limit benefit to cost ratio results.

A stagewise optimisation procedure has been used to estimate stationary values, local extrema, and the global extremum for each scenario.

4 Results and analysis

4.1 The base scenario

The scenario of diminishing value depreciation and constant annual maintenance costs forms the base scenario against which comparisons of other scenarios are made. Figure 2 shows the maximum and the lower limit benefit to cost ratios for the base scenario at each age at which full rehabilitation takes place when the housing stock is stationary and undergoes constant expansion at the rate of 2.0% per year ($r = 2.0$). Each set of maximum and lower limit benefit to cost ratios forms the boundaries of an envelope. All possible benefit to cost ratios for each expansion rate fall within each respective envelope. The benefit to cost ratios are given in units of dwelling service year equivalents per construction unit (sye/cu). The case of full rehabilitation at the age of zero represents the case of no rehabilitation because the corresponding maximum cost ratio is zero. Full results are listed in table 1 (see over).

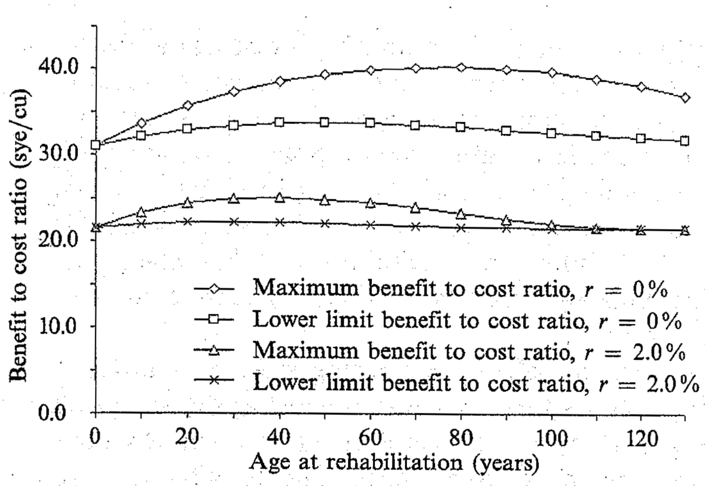

Figure 2. Maximum and lower limit benefit to cost ratios for base scenario ($r = 0$ and 2.0%).

Table 1. Results of (a) stationary and stable housing stock and (b) expanding and stable housing stock under base scenario.

Age at rehab (years)	Dwelling services ratio (sye/dwlg)	Cost ratio (cu per dwelling)			Benefit to cost ratio (sye/cu)		Maximum cost ratio (μ_y)
		replace	rehab	total	lower	maximum	
(a) *Stationary: new-build cost ratio* = 0.0000; *maintenance cost ratio* = 0.0100 *cu per dwelling.*							
0	0.544	0.0076	0.0000	0.0176	31.0	31.0	0.000
10	0.573	0.0071	0.0008	0.0178	32.1	33.6	0.112
20	0.592	0.0066	0.0014	0.0180	32.9	35.7	0.215
30	0.603	0.0062	0.0019	0.0181	33.4	37.3	0.308
40	0.609	0.0058	0.0023	0.0181	33.7	38.6	0.394
50	0.611	0.0055	0.0026	0.0181	33.7	39.4	0.473
60	0.609	0.0053	0.0029	0.0181	33.7	40.0	0.547
70	0.605	0.0051	0.0030	0.0181	33.5	40.2	0.615
80	0.600	0.0049	0.0032	0.0180	33.3	40.4	0.677
90	0.593	0.0048	0.0032	0.0180	33.0	40.1	0.733
100	0.587	0.0048	0.0032	0.0179	32.7	39.8	0.783
110	0.580	0.0049	0.0030	0.0179	32.4	39.0	0.826
120	0.573	0.0051	0.0028	0.0179	32.1	38.1	0.863
130	0.567	0.0054	0.0024	0.0178	31.8	36.9	0.894
(b) *Expanding: new-build cost ratio = 0.0202; maintenance cost ratio = 0.0100 cu per dwelling.*							
0	0.742	0.0043	0.0000	0.0345	21.5	21.5	0.000
10	0.786	0.0034	0.0022	0.0358	22.0	23.4	0.116
20	0.804	0.0027	0.0034	0.0363	22.2	24.4	0.224
30	0.808	0.0022	0.0039	0.0363	22.2	24.9	0.325
40	0.802	0.0019	0.0041	0.0361	22.2	25.0	0.423
50	0.793	0.0017	0.0040	0.0358	22.1	24.9	0.516
60	0.781	0.0017	0.0036	0.0355	22.0	24.5	0.606
70	0.770	0.0019	0.0031	0.0352	21.9	24.0	0.689
80	0.760	0.0023	0.0024	0.0349	21.7	23.4	0.762
90	0.752	0.0029	0.0016	0.0347	21.6	22.7	0.823
100	0.746	0.0036	0.0009	0.0346	21.6	22.1	0.871
110	0.743	0.0040	0.0004	0.0346	21.5	21.7	0.908
120	0.742	0.0042	0.0001	0.0345	21.5	21.6	0.936
130	0.742	0.0043	0.0000	0.0345	21.5	21.5	0.956

Note: dwlg, dwelling; max, maximum; rehab, rehabilitation; replace, replacement. For an explanation of the units see text, section 2.7.

4.2 Impact of expansion rate

Figure 2 shows that the level of the expansion rate of the housing stock has a greater impact upon benefit to cost ratios than does optimum timing or existence of rehabilitation. New-build construction forms a significant proportion of total costs that are not accompanied by a proportionate increase in benefits when a housing stock undergoes sustained growth. As a result, a stationary and stable housing stock can provide 44.2% more dwelling services per unit total cost than the housing stock which is doubling in size every 35 years (r = 2.0). When no rehabilitation takes place, the benefit to cost ratio of the expanding housing stock is 21.5 sye/cu compared with 31.0 sye/cu for the stationary housing stock.

The global extremum of 40.4 sye/cu occurs when the housing stock is stationary and full rehabilitation of entire dwelling cohorts takes place at no cost at the age of 80 years. Rehabilitation of New Zealand housing stock therefore cannot increase the quantity of benefits per unit total costs by more than 30.3% (31.0 to 40.4 sye/cu).

Actual increases would be less because full rehabilitation cannot be carried out at no cost.

4.3 Optimum timing and impact of rehabilitation

The benefit to cost ratio envelope shown in figure 2 is flatter and narrower when the housing stock undergoes expansion. The potential to provide a greater quantity of dwelling services per unit total cost as a result of full rehabilitation decreases as the expansion rate increases because replacement construction as a proportion of the size of the housing stock decreases regardless of whether rehabilitation takes place or not.

The optimum age at which to undertake full rehabilitation is 50 years when the housing stock is stationary. The lower limit benefit to cost ratio increases by 8.7% from 31.0 sye/cu for the case of no rehabilitation to 33.7 sye/cu. Greater increases in the benefit to cost ratios are possible if full rehabilitation takes place under budget.

The optimum age at which to undertake full rehabilitation is 30 years when the housing stock undergoes expansion ($r = 2.0$). The lower limit benefit to cost ratio increases by 3.3% from 21.50 sye/cu for the case of no rehabilitation to 22.2 sye/cu. Greater increases in the benefit to cost ratios are also possible if full rehabilitation takes place under budget. Potential increases in the benefit to cost ratio because of optimum timing of rehabilitation diminish when the expansion rate of the housing stock increases.

4.4 Sensitivity analysis of base scenario

Full rehabilitation may take place 10 years earlier or 10 years later than the optimum age without forfeiting more than 5% of potential increases in the lower limit benefit to cost ratios.

The simulation model is based on entire dwelling cohorts undergoing full rehabilitation. If a proportion only of dwelling cohorts undergo full rehabilitation, then increases in the lower limit benefit to cost ratio from optimum timing of full rehabilitation are scaled down by the proportion of those dwellings which undergo full rehabilitation. For example, if only 60% of a dwelling cohort undergoes full rehabilitation at the age of 50 years, then the 8.7% potential increase in the lower limit benefit to cost ratio of a stationary housing stock reduces to 5.2%.

Table 1 shows that annual maintenance costs form a substantial proportion of total costs. For example, annual maintenance costs of a stationary and stable housing stock form 55.3% of total costs when full rehabilitation takes place at the optimum age of 50 years. This proportion reduces to 27.7% when the expansion rate increases to 2.0% per year and full rehabilitation takes place at the optimum age of 30 years. Significant increases in the benefit to cost ratio can therefore be made if the costs of annual maintenance are reduced through better design and use of building materials and better management of maintenance. In the case of the stationary housing stock the local extremum benefit to cost ratio can increase by 5.9% (33.7 to 35.7 sye/cu) if annual maintenance costs are reduced by 10%. In the case of the expanding housing stock ($r = 2.0$) the local extremum benefit to cost ratio can increase by 3.0% (22.2 to 22.9 sye/cu) when annual maintenance costs are reduced by 10%. Potential increases in the benefit to cost ratio because of decreases in annual maintenance costs diminish when the expansion rate of the housing stock increases.

4.5 Realism of base scenario

The minimum benefit to cost ratio of 21.5 sye/cu for the base scenario occurs when the expansion rate is 2.0% per year and no rehabilitation takes place. The value of benefits to value of costs ratio that corresponds to this quantity of benefits to quantity of costs ratio is estimated to be 1.94 : 1 (see appendix C). The value of benefits therefore exceeds the value of costs for all cases considered under the base scenario.

The lower limit benefit to cost ratio for a stationary housing stock is optimised when dwellings undergo full rehabilitation at the age of 50 years and the maximum cost ratio is 0.473. A typical New Zealand dwelling can undergo full rehabilitation at the age of 50 years, including reroofing, rewiring, and upgrading of kitchen and bathroom within a budget of 47.3% of the cost to construct a new dwelling (Rawlinsons Group, 1996). The optimum age for full rehabilitation decreases from 50 years to 30 years when the expansion rate increases from zero to 2.0% per year. The corresponding maximum cost ratio decreases from 0.473 to 0.325. A typical New Zealand dwelling can also undergo full rehabilitation at the age of 30 years within a budget of 32.5% of the cost to construct a new dwelling. The maximum cost ratios used in the simulation model are realistic.

4.6 Straight-line depreciation

The dynamics and optimum timing of rehabilitation of the straight-line depreciation and constant annual maintenance costs scenario parallel those for the base scenario. The corresponding values for diminishing value depreciation are shown in brackets for comparison. A global extremum of 37.2 sye/cu (40.4 sye/cu) occurs when full rehabilitation takes place at the age of 80 years (80 years) and the annual expansion rate is zero (zero).

4.7 Increasing and decreasing annual maintenance costs with age

Figure 3 shows the lower limit benefit to costs ratios under the scenario of increasing annual maintenance costs with age (maintenance schedule 1), the base scenario of constant annual maintenance costs with age (maintenance schedule 2), and the scenario of decreasing annual maintenance costs with age (maintenance schedule 3). The housing stock is stationary and stable under each scenario and each scenario is based on diminishing value depreciation. Maximum cost ratios are estimated with a 6% real discount rate.

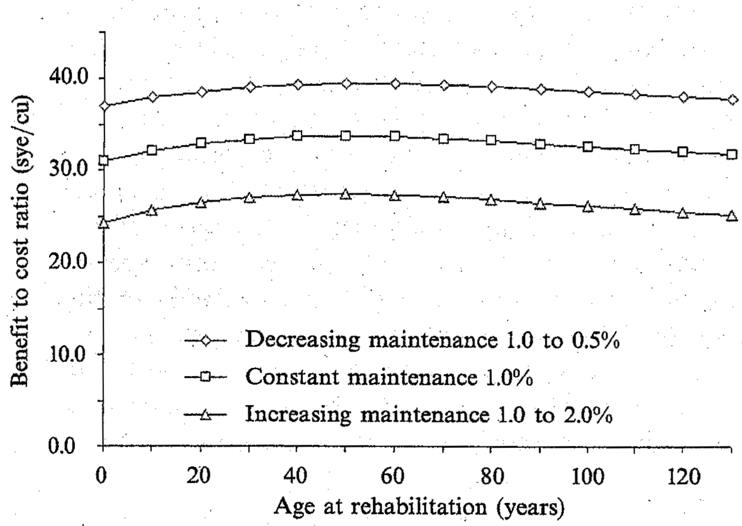

Figure 3. Lower limit benefit to cost ratios for scenarios based on decreasing, constant, and increasing annual maintenance costs with age ($r = 0\%$).

Optimum timing of rehabilitation occurs at the age of 50 years under each scenario. The benefit to cost ratios are larger if annual maintenance costs decrease with age and are smaller if annual maintenance costs increase with age. The local extremum under maintenance schedule 1 is 17.2% larger than that of the base scenario under maintenance schedule 2 (39.5 versus 33.7 sye/cu). The local extremum under maintenance schedule 3 is 18.7% smaller than that of the base scenario under maintenance schedule 2 (27.4 versus 33.7 sye/cu). Because benefits are the same under each scenario, a decrease in the benefit to cost ratio is caused by an increase in costs and vice versa. Although a larger budget on full rehabilitation is justified when maintenance costs increase with age, an increased expenditure on full rehabilitation to the full limit of the increased budget accounts for only 10.1% of the difference in the benefit to cost ratio under maintenance schedule 3 compared with that of the base scenario. The remaining difference is caused by increases in annual costs of maintenance with age.

Potential increases in the lower limit benefit to cost ratio through optimum timing of full rehabilitation are greater when increases in maintenance costs with age are more severe. The potential increase under the scenario of increasing maintenance costs with age is 12.8% compared with 8.7% under the base scenario. This potential increase of 12.8% reduces to 4.4% when the expansion rate increases from zero to 2.0% per year.

5 Discussion

A reduction in the expansion rate of New Zealand housing stock has a far greater impact than optimum timing or existence of rehabilitation. Almost 45% more dwelling services can be provided per unit total cost by a stationary and stable New Zealand housing stock compared with the same housing stock which doubles in size every 35 years. Not only are resources used more rapidly under conditions of sustained growth but also the use of resources is not optimised in the process. The reasons why are as follows.

The benefits of capital expenditure made over a current time interval are realised over current and future time intervals. Under conditions of sustained growth an expanding stream of capital expenditure is made before the benefits of capital expenditure made at the start of the stream are fully realised. In contrast, the future benefits of capital expenditure that replaces dwelling losses from a stationary and stable housing stock over a current time interval are effectively realised over the same time interval. This is because the dwelling services provided by each dwelling cohort within a stationary and stable housing stock over the current interval mirror the dwelling services provided by an individual dwelling cohort over future time intervals.

Whether New Zealand housing stock continues to expand or not is contingent on New Zealand immigration policies. Positive net migration currently forms the major source of effective demand to form additional households (New Zealand Department of Statistics, 1996). This is because the fertility of the natural population has declined since the 1960s to the extent that the natural population now barely replaces itself. The average number of persons per household has declined from over 6 at the turn of the century to 2.8 at the last Census in 1991 (New Zealand Department of Statistics, 1996). Further decreases over the next number of decades are likely to be gradual.

Existing immigration policies and, to a lesser extent, rural to urban drift will ultimately lead to a need to extend existing infrastructure in the major centres, the cost of which will be carried by the entire community through national and local taxation. One argument for increasing the population of New Zealand is that based on the benefits which can result from economies of scale. But all costs, as well as benefits, should be taken into account. Many of the costs of sustained growth are externalities; such as social costs outlined by Mishan (1967) and environmental costs outlined by Pearce and Turner (1990). Another argument is that based on altruism.

The population density of New Zealand is far less than that of many countries and, according to the argument, New Zealand should carry a share of the burden of overcrowding in undeveloped countries. An opposing 'lifeboat ethics' argument has been presented by Hardin (1977). New Zealand immigration policies are unlikely to change dramatically over the next number of decades. Current levels of positive net migration are likely to continue and, unless these levels increase in proportion to the size of the total population, the growth rate of the total population will gradually decline as will the growth rate of the housing stock.

If full rehabilitation of New Zealand dwellings takes place between the ages of 30 years and 50 years, then a greater quantity of dwelling services can be provided per unit total cost through reductions in the replacement rate. Expenditure on full rehabilitation should not exceed that which can be justified on an actuarial basis, whether this expenditure is for a group of dwellings or an individual dwelling and whether government, local government, or a private individual carries the cost. Bank loans for the purposes of home improvements are readily available to those who are currently earning an income. The one sector of the community which could benefit from increased access to home improvement loans is that of the elderly. For many, their home represents their sole store of savings. Rather than allow these houses to sink into disrepair, a loss that is ultimately a national loss, reverse mortgages for the purposes of home improvements could be made available to the elderly. Reverse mortgages could also fulfil the role of a superannuation scheme. Government may be required to take a lead by underwriting the risks of reverse mortgages.

The costs of annual maintenance form a significant proportion of the total costs required to sustain the housing stock and reductions in these costs offer greater potential to optimise the use of resources than does optimum timing of full rehabilitation. The simulation model assumes that full rehabilitation of entire dwelling cohorts can take place within a maximum justifiable budget. Although the maximum budgets used in the model are realistic for a typical dwelling, what is currently not known is the distribution of the quality of dwellings within each dwelling cohort. Not all dwellings can and should undergo full rehabilitation when greater benefits can be realised by undertaking demolition and replacement or redevelopment. In contrast, the depreciation of all dwellings can be retarded through appropriate and timely maintenance. A random survey of 400 New Zealand dwellings revealed that deferred maintenance averaged NZ$3200 per dwelling (Page et al, 1995), or almost three times the average costs of annual maintenance. Homeowners need to be better educated as to the consequences of deferred maintenance and the value and importance of appropriate and timely maintenance.

The simulation model can be used to forecast levels of activities within the construction industry. For example, replacement construction currently forms less than 20% of all new construction. This proportion will increase as the expansion rate declines until all new construction is replacement construction by the time the housing stock is stationary. When the expansion rate of New Zealand housing stock eventually declines to 1.0% per year, a rate that represents a doubling in size of the housing stock every 70 years, replacement construction would eventually form almost 40% of new construction. Factors of obsolescence would have a greater influence on the housing market.

The simulation model can be easily adapted to model the dynamics of the energy and mass flows required to sustain housing stock in particular, or building stock and infrastructure in general. Estimates of direct and indirect energy enable forecasts of the CO_2 contribution to the atmosphere by the construction industry. Estimates of the direct mass flows of building materials or the embodied mass flows of minerals, metals, and fresh water required to sustain building stock and infrastructure highlight the need for and facilitate forward planning of the use of these resources.

The simulation model is based on assumed schedules of depreciation and annual maintenance costs over the full economic life span of dwellings. Although the results of the model confirm that these assumptions are realistic, longitudinal studies of these factors should nonetheless be carried out and the results used within the simulation model to ensure greater precision of forecasts. The set of assumptions that underpin the model can and should be modified or relaxed. For example, full rehabilitation is assumed to take place once only whereas, in practice, partial rehabilitation takes place at periodic intervals.

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APPENDIX A

Introduction to life tables

A life table is a dynamic simulation model comprised of a set of nonlinear schedules that are constructed from age-specific loss and survivorship data. Each schedule within a life table is a mathematical transform of every other as follows (Keyfitz, 1968).

Probability of loss (${}_nq_x$) is the proportion of dwellings that are standing at the beginning of an age interval that will be lost from the housing stock before reaching the end of the age interval. Probability of loss is given by

$$ {}_nq_x = \frac{l_x - l_{x+n}}{l_x} , \tag{A1}$$

where l_x and l_{x+n} are the expected surviving stock which survive to the exact age x and $(x+n)$ marking the beginning of each respective age interval. By convention, the subscript for the age interval (n) is omitted when $n = 1$.

Stock losses (${}_nd_x$) give the dwelling losses over successive age intervals x to $(x+n)$. Stock losses are given by

$$ {}_nd_x = l_x - l_{x+n} . \tag{A2}$$

The stock in age interval (${}_nL_x$) gives the number of dwellings in each age interval within a stationary and stable housing stock. The stock in age interval can also be interpreted as the number of dwelling service years between the start of two adjacent age intervals x and $(x+n)$ provided by a single dwelling cohort. The stock in age interval is given by

$$ {}_nL_x = nl_x - {}_nd_x \, {}_na_x , \tag{A3}$$

where ${}_na_x$ is the average number of dwelling service years that are provided by dwellings lost over the age interval.

Total useful life (T_x) gives the number of stationary stock dwellings in an indicated age interval and all subsequent age intervals. Alternatively, total useful life is the total dwelling service years provided by a dwelling cohort after the age of x. Total useful life is given by

$$ T_x = {}_nL_x + {}_nL_{x+n} + {}_nL_{x+2n} + \ldots = \sum_{k=0}^{\infty} {}_nL_{x+kn} , \qquad |k = 0, 1, 2, \ldots| . \tag{A4}$$

Life expectancy at age x (e_x), gives the average number of dwelling service years remaining to be provided by those dwellings which are still standing at the beginning of an age interval x and $(x+n)$. Life expectancy at age x, or remaining average economic life at age x, is given by

$$ e_x = \frac{T_x}{l_x} . \tag{A5}$$

The life expectancy upon entry (e_0), or average economic life (upon entry), is given by

$$ e_0 = \frac{T_0}{l_0} . \tag{A6}$$

APPENDIX B

Benefits and costs need not be in the same metric when using the benefit to cost ratio criterion to rank alternative investments. This demonstration is based on Fisher (1923). The conventional metric of money serves as a measure of value. The concept of value depends on price, that of price in turn on exchange, and that of exchange on transfer. Value is found by multiplying a quantity by its price. For example,

$$\frac{\text{value of benefits}}{\text{value of costs}} = \frac{V_\text{b}}{V_\text{c}} = \frac{\text{price of benefits} \times \text{quantity of benefits}}{\text{price of costs} \times \text{quantity of costs}} = \frac{P_\text{b} Q_\text{b}}{P_\text{c} Q_\text{c}} = K\frac{Q_\text{b}}{Q_\text{c}} . \tag{B1}$$

Consider two investments where the first provides $V_{\text{b}1}$ benefits for $V_{\text{c}1}$ costs and the second provides $V_{\text{b}2}$ benefits for $V_{\text{c}2}$ costs so that

$$\frac{V_{\text{b}1}}{V_{\text{c}1}} = K\frac{Q_{\text{b}1}}{Q_{\text{c}1}}, \quad \text{and} \quad \frac{V_{\text{b}2}}{V_{\text{c}2}} = K\frac{Q_{\text{b}2}}{Q_{\text{c}2}} . \tag{B2}$$

The quantity of benefits to quantity of costs criterion gives the same rankings as the value of benefits to value of costs ratio, provided the prices P_b and P_c are not vectors. Because maintenance and rehabilitation costs in the simulation model are first estimated as ratios of the value of maintenance and rehabilitation to the value of construction respectively and then converted into proportions of dwelling construction units, the requirement that prices P_b and P_c not be vectors is therefore satisfied.

APPENDIX C

The quantity of benefits to quantity of costs ratio of 21.5 sye/cu is estimated to be equivalent to the value of benefits to value of costs ratio of 1.94:1. This estimate is based on data for Birkdale which is a middle-income suburb located in Auckland. The size and quality of dwellings in Birkdale are typical of those to be found throughout New Zealand.

$$\frac{\text{value of benefits}}{\text{value of cost}} = \frac{21.5 \text{ sye}}{1 \text{ cu}} \frac{(\$260/\text{week} \times 52 \text{ weeks} \times 0.60)/\text{sye}}{(100\text{m}^2 \times \$900/\text{m}^2)/\text{cu}} = \frac{1.94}{1}, \tag{C1}$$

where typical cost of renting a dwelling is NZ\$260 per week (*The New Zealand Herald* 1966); typical improvement to capital value ratio is 0.60 (Valuation New Zealand, 1996); typical cost of constructing a dwelling is NZ\$900 m^{-2} (Rawlinsons Group, 1996); typical size of dwellings is 100 m^2 (Nana, 1979).

The value of benefits far exceeds the value of costs.