



Energy and mass flows of housing: estimating mortality

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Abstract

Stock and flow models of housing should be driven by empirical schedules of mortality. A selection of analytic, life table, and stock and flow models used to estimate the mortality of housing are examined. The assumptions which underpin each of these models, data difficulties, issues of validation and misapplication of mortality indicators are addressed. A stock and flow model is used to estimate the energy and mass flows required to sustain dwelling services in the companion paper (Johnstone IM. Energy and mass flows of housing: a model and example. *Building and Environment* 2000;36(1):27–41). © 2000 Elsevier Science Ltd. All rights reserved.

1. Introduction

Stock and flow models of housing are used to simulate the dynamics of benefits in the form of dwelling services and the costs of sustaining those services — new-build construction which adds to the size of a housing stock, maintenance, rehabilitation, demolition and replacement construction. Such models should be driven by empirical schedules of mortality otherwise the precision and subsequent usefulness of these models is limited. This paper examines a selection of analytic, life table and stock and flow models that can be used to estimate the mortality of housing. The assumptions which underpin each of these models, data difficulties, issues of validation and misapplication of mortality indicators are addressed.

2. Analytic model of mortality

Mortality pertains to probability of loss, survivorship and life expectation. Measures of mortality include average life expectancy upon entry and life span. These measures are more appropriately referred to as average service life and service life span when applied to housing. There is a distinction between the service life and the economic life of a dwelling. The

service life of a dwelling is that period between entry to and departure from a housing stock during which a dwelling provides dwelling services. The economic life of a dwelling reaches an end when the market value of the property plus the costs of demolition and clearing the site are less than the value of the cleared site in a new and higher use. A dwelling can continue to be technically efficient beyond its economic life by earning net annual returns in the form of rent or imputed rent.

The service life span (ω) of a housing stock is hereby defined to be that age beyond which less than 0.1% of the oldest dwelling cohort still stands and provides dwelling services. This definition based on Shryock et al. [1] enables sensible comparisons of the service life spans of different housing stocks as exceptional dwellings are excluded.

Estimates of the mortality of housing stock that are sufficiently precise so as to be useful for forecasting purposes, involve solving hundreds of non-linear equations. This task is beyond the capabilities of an analytic model, but not that of a simulation model. Analytic models are useful under restricted conditions such as when the equations describing a process are linear and when there are relatively few simultaneous equations to be solved. In order to satisfy these restrictions, analytic models are frequently an over-simplification of the process being modelled. An example is

Needleman's [2] analytic model of the mortality of British housing stock.

Needleman developed an analytic model to estimate the 'normal' life of British dwellings, the normal life defined by Needleman as being the number of years that elapse before half the dwellings built in a particular year have been demolished. Needleman estimated the normal life of British housing stock to be 141.7 years based on an average expansion rate of 1.3% per year over the previous 160 years and an average replacement rate of 0.25% per year between 1881 and 1961 [2]. This estimate can be tested for precision because the normal life, average service life and service life span of dwellings in Needleman's model are, for all practical purposes, one and the same [3]. A number of British dwellings of traditional construction remain in use after 400 years [4] and Riley [5] estimated that 0.1% of the British housing stock had been constructed prior to 1700. A service life span of 141.7 years is therefore a gross underestimate.

The lack of precision in Needleman's analytic model is due primarily to the unsubstantiated assumption of mortality built into the model. Needleman explicitly assumed that only the oldest dwellings within a housing stock are demolished and implicitly assumed that all dwellings are subject to the same regime of mortality. The service life of a dwelling is limited by the physical life of its structural system. Isolated rotting piles, bearers, floor joists, studs, rafters and the like can be replaced, but when general structural failure occurs replacement of the structural system involves dismantling the entire dwelling and starting anew. However, few dwellings are demolished due to general failure of its structural system. Most departures of dwellings from a housing stock are the end result of an economic process and the potential service life of most dwellings is not realised. At some stage during the service life of a dwelling, a replacement dwelling, a change in use from residential to commercial, or an alternative use of the site is expected to provide a greater discounted stream of net income to the owner or to the community. Even abandonment is the result of an economic decision to discontinue occupancy and forgo ownership.

A number of empirical studies of the mortality of housing stock confirm that dwelling losses occur at all ages and that these losses are non-linear over the service life span of dwelling cohorts. In a pioneering study of the mortality of a sample of Indianapolis housing stock, Gleeson [6] estimated that of each original dwelling cohort which enters the housing stock, approximately 14% are lost between entry and the age of 70 years with a further 29% being lost between the age of 70 and 95 years. Johnstone [7] estimated similar findings for New Zealand housing stock. In a study of Japanese timber dwellings, Komatsu et al. [8] esti-

ated that approximately 15% of original dwellings are lost between entry and the age of 20 years with a further 35% being lost between the age of 20 and 40 years.

3. Life table models of mortality

A life table is a simulation model that estimates the mortality of a stationary and stable population or an individual cohort within a population. The size of each cohort as a proportion of the size of the total population remains constant over time when the population is stable. Variations and extensions of the theories and life table models of classical population dynamics have been developed and applied by actuaries [9], demographers [10], ecologists [11] and engineers over the past 200 years.

3.1. Construction of a life table

A life table of housing stock comprises a set of non-linear schedules that are constructed from age-specific loss and survivorship data. The age-specific loss rate (M_x) of dwellings departing from a housing stock is given by

$$M_x = \frac{D_x}{P_x} \quad (1)$$

where P_x is the size of a dwelling cohort of age between x and $(x + 1)$ standing at the middle of a calendar year and D_x is the age-specific dwelling losses from this dwelling cohort over the calendar year.

In proceeding from statements of past age-specific loss rates to statements of probability regarding future loss, an implicit assumption is made that the future mortality of a housing stock will be the same as that in the past. A dwelling cohort which has an average age of $x + 1/2$ at midyear has an average age of x at the beginning of the year. If age-specific dwelling losses (D_x) are distributed uniformly throughout the year so that one half of departures occur during the first half of the year, then the dwelling cohort standing at the beginning of the year would be $P_x + 1/2D_x$. The annual probability of loss (q_x), or the probability that any one dwelling of age x standing at the beginning of a calendar year is lost from the housing stock during that year, is expressed by

$$q_x = \frac{D_x}{P_x + 1/2D_x} = \frac{M_x}{1 + 1/2M_x} \quad (2)$$

Each schedule of a life table is a mathematical transform of one another as follows. Probability of loss (nq_x) is the proportion of dwellings that are standing at the beginning of an age interval that will be lost

Table 1
Typical abridged life table of housing stock

1. Age interval (years) $(x) - (x + n)$	2. Probability of loss $({}_nq_x)$	Of 100,000 dwellings		Stationary stock		
		3. Surviving stock at start of interval (l_x)	4. Stock losses over interval $({}_nd_x)$	5. Stock in age interval $({}_nL_x)$	6. Total useful life at start of interval (T_x)	7. Average service life at start of interval (e_x)
0–5	0.00001	100000	1	499997	8999990	90.0
5–10	0.00004	99999	4	499985	8499993	85.0
10–15	0.00010	99995	10	499950	8000008	80.0
15–20	0.00024	99985	24	499865	7500058	75.0
20–25	0.00054	99961	54	499670	7000193	70.0
25–30	0.00113	99901	113	499252	6500523	65.1
30–35	0.00223	99794	223	498412	6001271	60.1
35–40	0.00419	99571	417	496812	5502859	55.3
40–45	0.00741	99154	735	493932	5006047	50.5
45–50	0.01245	98419	1225	489032	4512115	45.8
50–55	0.01984	97194	1928	481150	4023083	41.4
55–60	0.03011	95266	2868	469160	3541933	37.2
60–65	0.04362	92398	4030	451915	3072773	33.3
65–70	0.06055	88368	5351	428462	2620858	29.7
70–75	0.08086	83011	6713	398302	2192396	26.4
75–80	0.10429	76304	7958	361625	1794094	23.5
80–85	0.13041	68346	8913	319447	1432469	21.0
85–90	0.15872	59433	9433	273582	1113022	18.7
90–95	0.18866	50000	9433	226417	839440	16.8
95–100	0.21971	40561	8913	180552	613023	15.1
100–105	0.25141	31654	7958	138375	432471	13.7
105–110	0.28330	23696	6713	101697	294096	12.4
110–115	0.31508	16983	5351	71537	192399	11.3
115–120	0.34646	11632	4030	48085	120862	10.4
120–125	0.37727	7602	2868	30840	72777	9.6
125–130	0.40127	4134	1928	18850	41931	8.9
130–135	0.43656	2806	1225	10967	23087	8.2
135–140	0.46490	1581	735	6067	12120	7.7
140–145	0.49291	846	417	3187	6053	7.2
145–150	0.51981	429	223	1587	2866	6.1
150–155	0.54854	206	113	747	1279	6.2
155–160	0.58065	93	54	330	532	5.7
160–165	0.61538	39	24	135	202	5.2
165–170	0.66667	15	10	50	67	4.5
170–175	0.80000	5	4	15	17	3.4
175–180	1.00000	1	1	2	2	2.0

from the housing stock before reaching the end of the age interval. Probability of loss is given by

$${}_nq_x = \frac{l_x - l_{x+n}}{l_x} \quad (3)$$

where l_x and l_{x+n} are the expected surviving stock which survive to the exact age x and $(x + n)$ marking the beginning of each respective age interval. By convention, the subscript for the age interval (n) is omitted when $n = 1$.

Stock losses (${}_nd_x$) give the dwelling losses over successive age intervals x to $x + n$. Stock losses are given by

$${}_nd_x = l_x - l_{x+n} \quad (4)$$

The stock in age interval (${}_nL_x$) gives the number of

dwellings in each age interval within a stationary and stable housing stock. The stock in age interval can also be interpreted as the number of dwelling service years between the start of two adjacent age intervals x and $(x + n)$ provided by a single dwelling cohort. The stock in age interval is given by

$${}_nL_x = {}_n l_x - {}_n d_{x+n} a_x \quad (5)$$

where na_x is the average number of dwelling service years that are provided by dwellings lost over the age interval.

Total useful life (T_x) gives the number of stationary stock dwellings in an indicated age interval and all subsequent age intervals. Alternatively, total useful life is the total dwelling service years provided by a dwelling cohort after the age of x . Total useful life is given

by

$$T_x = {}_nL_x + {}_nL_{x+n} + {}_nL_{x+2n} + \dots = \sum_{k=0}^{\infty} {}_nL_{x+kn} \quad (6)$$

for $k = 0, 1, 2, \dots$

Life expectancy at age x (e_x), gives the average number of dwelling service years remaining to be provided by those dwellings which are still standing at the beginning of an age interval x and $(x+n)$. Life expectancy at age x , or remaining average service life at age x , is given by

$$e_x = \frac{T_x}{l_x} \quad (7)$$

The life expectancy upon entry (e_0), or average service life (upon entry), is given by

$$e_0 = \frac{T_0}{l_0} \quad (8)$$

Dwelling entries exactly match and replace total dwelling losses of all ages over each time interval t to $(t+n)$ when a housing stock is stationary:

$$l_0 = \sum_{x=0}^{\omega} {}_n d_x \quad (9)$$

The average service life of a stationary and stable housing stock is given by the inverse of its annual replacement rate expressed as a proportion of the size of the total housing stock. For example, if the annual replacement rate is 1.0% per year, then the average service life of a housing stock is 100 years. This relationship hold true, regardless of the distribution of dwelling losses from each dwelling cohort over its service life span, but only when the housing stock is stationary and stable.

3.2. Classification as complete or abridged life table

Life tables are classified as being complete or abridged. A complete life table contains data for every single year of age from 0 until ω , the service life span of the housing stock. An abridged life table contains data by age intervals of 5 or 10 years. An example of an abridged life table for housing stock is shown in Table 1. The layout and notation are identical for that of a human population. Column 1 gives the age interval between two exact ages x and $(x+n)$. The age interval (n) of the abridged life table is five years. The radix of the life table (l_0), or initial dwelling entries at age 0, is set at 100,000 dwellings by convention.

3.3. Current and generation life tables

There are two types of life tables, namely the current or period life table and the generation or cohort life table. Both models use age-specific data on dwelling losses and survivorship. A current life table uses cross-sectional data whereas a generation life table uses longitudinal data. A current life table does not represent the mortality experienced by an actual cohort over its service life but is instead a synthetic construct of a hypothetical cohort that provides a 'snapshot' summary of mortality at an instant in time. An implicit assumption is made that all dwelling cohorts are subject to the same regime of mortality. In other words, the housing stock is assumed to be subject to static mortality as opposed to dynamic mortality where dwelling cohorts are subject to different regimes of mortality. A series of generation life tables enables detection of dynamic mortality and general shifts in mortality over time.

3.4. An application of a current life table

Gleeson [6] and Komatsu et al. [8] respectively used a current life table approach to estimate the mortality of a sample of Indianapolis housing stock and light-weight timber-framed dwellings in Japan. Gleeson used demolition statistics as a measure of dwelling losses, which minimises potential double counting, but excludes dwelling losses due to mergers or changes in use. The year of construction was unavailable for 12% of the sample of 175,174 dwellings. Gleeson allocated these 'age unknown' dwellings to cohorts in the proportion of the distribution of dwellings with known age and used grouping methods to minimise the effects of data errors and idiosyncratic fluctuations due to the small size of the sample of dwellings. Data on dwelling losses collected for the two separate years of 1979 and 1980 were averaged and dwellings constructed over the interval 1905 to 1914 were grouped together.

Gleeson conceded that the method he used to smooth out fluctuations caused by data errors and small samples was crude but, in his opinion, use of more elaborate and sophisticated methods in estimating housing mortality was best deferred for more refined studies that are able to use improved data. Because there was a scarcity of data for very old dwellings, Gleeson fitted a Gompertz curve tail to the survivorship schedule of the Indianapolis life table at the ages 75–95 years to estimate survivorship for dwellings older than 95 years. A Gompertz curve closely approximates the survivorship curve for human populations over the latter years of life [12]. Gleeson estimated the average service life of the sample of Indianapolis housing stock to be 99.6 years from the resulting life table.

By fitting a Gompertz curve to the tail of the survivorship schedule within the Indianapolis life table, Gleeson effectively extrapolated 57% of the sum total of dwelling losses over the remaining service life span of the life table. Uncertainty filters throughout the entire life table as a result. Although the use of a Gompertz curve has proved to be a useful approximation for estimating human mortality, the same does not necessarily apply for housing. There are numerous curves that could have been used to extrapolate the Indianapolis survivorship curve from the age 95 onwards. Using data in Gleeson's paper, Johnstone [13] estimated the true average service life of Indianapolis housing stock to lie between 96 and 118 years on the basis that the tail of the true survivorship curve lies somewhere within a zone defined by the boundaries of a fitted straight line and an exponential curve (see Fig. 1). The remaining average service life at the beginning of each successive age interval becomes progressively more indeterminate. For example, the remaining average service life at the age of 75 years lies between 29 and 56 years and the remaining average service life at the age of 150 years lies between 0 and 57 years. Gleeson [14] has since used the 1985 Indianapolis life table to develop an actuarial model which estimates the maximum cost at which renovation is preferred to new construction for each age at which renovation takes place. The level of uncertainty and resulting imprecision built into the 1985 Indianapolis life table negates its use for decision-making.

4. Stock and flow model of mortality

If age-specific data on dwelling losses and survivorship are unavailable, then the mortality of housing stock can be estimated by using test schedules of mor-

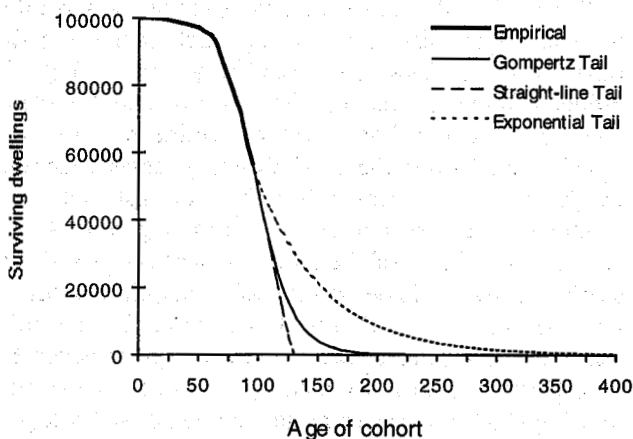


Fig. 1. Surviving stock schedule of Indianapolis housing stock showing the boundaries of the envelope of uncertainty.

tality within a stock and flow model of mortality, provided suitable alternative data is available against which the results of the model can be validated. This is the approach that Johnstone [7] adopted to estimate the mortality of New Zealand housing stock. Nauda et al. [15] and Brewer [16] have promoted the use of Control Theory and the Kolmogorov partial differentiation equation, respectively, to model the dynamics of a housing stock. These models, however, do not provide the same degree of transparency and simplicity of construction as that offered by a model based on classical population dynamics. A current life table model cannot be used to simulate the dynamics of an expanding housing stock because it is restricted to simulating replacement construction only within a stationary and stable housing stock and all state variables are fixed. Johnstone's [7] stock and flow model of mortality consists of a linked series of longitudinal probability of loss and survivorship schedules that, in combination, simulate the flows of replacement and new-build construction within an expanding housing stock. State variables can change so the model can simulate either static or dynamic mortality. The stock and flow model of mortality is similar in structure to the dynamic energy and mass flow model described in the companion paper [17].

4.1. Test schedules of mortality

Johnstone [7] used test schedules of mortality based on a search plane of possible distributions of dwelling losses from a dwelling cohort over its service life span. Test schedules of mortality based on models of decay in dwelling components, combined with maintenance and rehabilitation strategies, can also be used. For example, Wyatt [18] based his conceptual model of British housing stock on a multi-deck stack where each stack represents an equal period of time. Dwellings pass down the stack at a rate determined by decay curves based on a conceptualisation of decay in dwelling components. Decisions are made at each deck as to whether rehabilitation should take place or not. Dwellings that undergo rehabilitation return up the stack, the re-entry point being dependent on which upgrading strategy has been adopted. Each dwelling ultimately reaches the bottom of the stack where demolition takes place.

Replacement of each typical building in Kohler et al.'s [19] model of the energy and mass flows of the German building stock is determined by a combination of stochastic modelling of the lifetime of building elements and a maintenance and refurbishment strategy. A building becomes vacant if it is not refurbished by a certain time and it is demolished if it stays vacant for a certain time. Buildings of the same typology therefore depart from the building stock over a range of

ages. The energy and mass flow model effectively assumes regimes of mortality for each class of buildings. These assumed regimes of mortality have not been validated due to the lack of suitable data on survivorship and losses.

4.2. Dynamic mortality

By using a test schedule of mortality approach, Johnstone [7] was able to establish that the mortality of New Zealand housing between 1860 and 1980 had been dynamic. The probability of loss of dwellings from each cohort had simultaneously increased and decreased with each increase and decrease in the expansion rate of the housing stock. Each dwelling cohort had been subject to a different regime of mortality as a result. Fig. 2 shows the distribution of dwelling losses from different dwelling cohorts. The initial size of each dwelling cohort is scaled to 100,000 dwellings, the radix of a life table.

A number of forecasts of annual replacement rates based on extrapolation of past dwelling losses implicitly assume that total dwelling losses of all ages increase as the size of a housing stock increases. Under conditions of dynamic mortality, total dwelling losses can actually decrease when the size of a housing stock increases monotonically.

The probability of loss function $P(x,r)$ for New Zealand housing is given by the product of a base probability of loss (q_x) and a multiplier function:

$$P(x,r) = q_x(1 - 78.62r)^{0.70} \quad (10)$$

where r is the annual expansion rate of the housing stock.

The classical concept of life expectancy is based on future mortality being the same as that in the past.

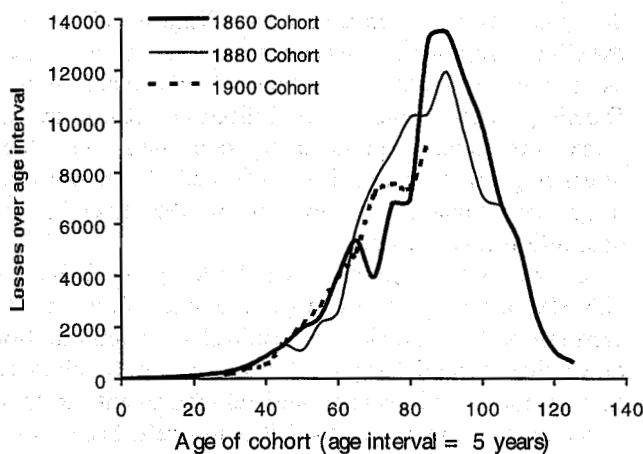


Fig. 2. Distribution of dwelling losses from New Zealand dwelling cohorts scaled to 100,000 dwelling entries.

The concept of dynamic life expectancy requires explicit assumptions as to what the expansion rate of a housing stock will be in the future. Given that the base mortality and multiplier function which applied to the New Zealand housing in the past remains the same in the future, then the current average service life of 90 years would increase to 130 years should the expansion rate of the New Zealand housing stock decline to zero. The severity of the base mortality may decrease in the future due to the use of more durable building materials and increases in the level and number of cycles of rehabilitation undertaken within the housing stock. The multiplier function may decrease in the future due to decreasing opportunities for in-fill housing and housing policies that reduce the scale of demolition and redevelopment during expansionary cycles.

4.3. Validation of test schedules of mortality

The availability of field survey data will determine which test, or combination of tests, can be used to validate test schedules of mortality. For example, field survey data may be restricted to a series of the size of the housing stock, a single distribution of dwellings by age (stock profile), and a series of total dwelling losses (of all ages) over successive time intervals as was the case with New Zealand housing stock.

A series of total dwelling losses over successive time intervals can be estimated using the residual method which makes use of annual data series of new dwelling completions and censi statistics on the number of dwellings in the housing stock. Merrett [20] has pointed out that uncertainties in the data series are amplified in the residual. Total dwelling losses can be estimated with greater precision using the residual method when the expansion rate of a housing stock declines because replacement of total dwelling losses then form a greater proportion of all new dwelling entries.

The ability of a stock and flow model of mortality to replicate the size of a housing stock over successive time intervals does not necessarily imply that the model exhibits precision. Net additions and replacement of total dwelling losses determine the size of a housing stock, but the total dwelling losses generated by the model may not reflect the true distribution of dwelling losses by age. A more exacting test of precision is the ability of the model to replicate a series of stock profiles at single or successive snapshot points in time. However, a stock profile test alone is an insufficient test of precision because stock profiles generated by different regimes of mortality tend to merge when a housing stock undergoes rapid expansion. A stock profile test should be used in conjunction with a test of the ability of the model to replicate the true magnitude of total dwelling losses. When a housing stock under-

goes expansion, different distributions of dwelling losses over the service life span of dwelling cohorts result in different annual replacement rates because dwelling losses from younger, and hence larger, dwelling cohorts tend to predominate. Total dwelling losses are greater when a larger proportion of dwelling losses from each dwelling cohort occur early during its service life span. The extent of these differences is illustrated as follows.

Take two housing stocks of the same size and the same average service life of 75 years but which are exposed to different regimes of mortality. Dwelling losses from each dwelling cohort within the first housing stock remain constant over successive age intervals while the distribution of dwelling losses from each dwelling cohort within the second housing stock follow that of a normal curve. If both housing stocks expand at a constant rate of 2.0% per year over a period sufficiently long enough for each respective stock profile to be proportionately stable, then total dwelling losses from the first housing stock are in the ratio of 1.5:1 to that from the second.

A high goodness-of-fit between simulated data and field survey data is not to be unexpected when a housing stock undergoes expansion and the coefficient of determination (R^2) statistic may not adequately differentiate between test schedules of mortality whose performance range from average to accurate. Appropriate statistical methods for survival data analysis are set out in Lee [21]. Type II errors are particularly important in goodness of fit testing when assessing the accuracy of a test schedule of mortality. Fotheringham and Knudsen [22] recommend a significance level of $\alpha=0.25$ whenever increasing values of the goodness-of-fit statistic indicate decreasing model accuracy.

Uncertainty in validation data negates establishing sufficiently precise schedules of mortality for short-

range forecast purposes. Nonetheless, a best fit schedule of mortality can decisively establish whether a housing stock is subject to static or dynamic mortality and can be used to generate realistic dynamics of a housing stock for the purposes of better insight and understanding.

5. Rough estimates of average service life based on limited data

Rough estimates of the average service life of a housing stock can be made when a series of data is limited, provided census statistics of the size of the housing stock and a recent survey of the housing stock by age of construction (stock profile) are available. For example, the age of the oldest surviving dwellings in a housing stock may be 200 years. The average service life of the housing stock therefore lies somewhere between 200 years, the service life span of the housing stock, and the lower limit of zero years. The range of uncertainty is reduced as follows.

Net entries (new-build construction) to a housing stock over the interval between census form a lower-limit estimate of gross entries (new-build construction plus replacement construction). Annual entries, or dwelling cohorts, are estimated by assuming a constant expansion rate between census. The precision of an estimate increases when the expansion rate of a housing stock is high because replacement construction then forms a smaller proportion of total new construction.

A recent stock profile enables an estimate of the proportion of dwellings lost from a dwelling cohort since entry to the housing stock. Census statistics on the size of the housing stock should date back for a century, if possible, so that sufficient time has elapsed to allow a sizeable proportion of dwellings to have departed from each dwelling cohort. Survivorship of a dwelling cohort is then interpolated by assuming a straight-line decline in survivorship from 100% at entry to the current survivorship. This is followed by a second assumed straight-line decline in survivorship to 0% by the end of the projected service life span of the cohort. A life table can then be constructed from the ensuing survivorship curve.

Fig. 3 shows an example of the survivorship schedule of a dwelling cohort where 40% of dwellings have departed by the age of 100 years. The original size of the dwelling cohort is estimated from net entries and is scaled to 100,000 dwellings, the radix of a life table. The projected service life span of the dwelling cohort is 200 years, the current service life span of the housing stock. A rough estimate of the average service life of the dwelling cohort is 110 years. If replacement of total dwelling losses form 10% of net entries to the housing stock, then the true size of the original dwell-

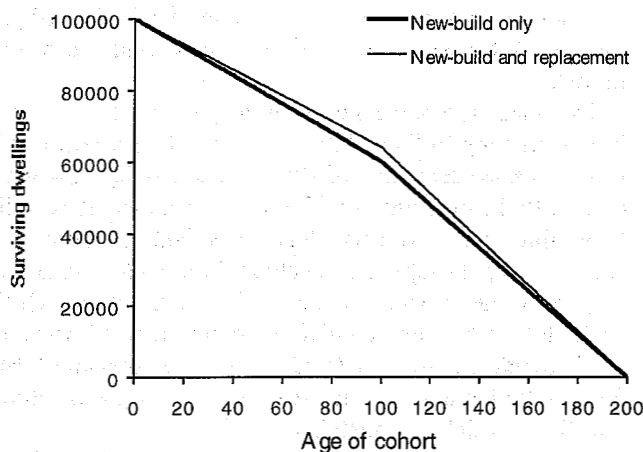


Fig. 3. Survivorship schedule of dwelling cohorts based on limited data.

ing cohort is under-estimated by 10%, and 36% of dwellings in the original dwelling cohort have departed by the age of 100 years. A more precise estimate of the average service life of the dwelling cohort is 114 years.

If the mortality of a housing stock is static, then estimates of the average service life of the housing stock can be made with greater precision by constructing a survivorship schedule of a synthetic dwelling cohort using data on succeeding dwelling cohorts. Survivorship within the synthetic schedule should decrease monotonically with age otherwise there would be a reversal in mortality. Any increase in survivorship with age would be a clear signal that the mortality of the housing stock is not static.

6. Misapplication of mortality indicators

Indicators of mortality are frequently misapplied. For example, in a study of housing in England, Meikle and Connaughton [23] state that a very low annual replacement rate of 0.1% between 1986 and 1990 implies that houses in England will need to last up to 1000 years on average. The housing stock had increased by an average annual expansion rate of 0.9% per year over the same period (18,852,000 dwellings in 1986 to 19,725,000 dwellings in 1991) [23]. Meikle and Connaughton based their estimate on an inverse relationship between the average service life and the annual replacement rate of a housing stock which applies only for a stationary and stable housing stock. By doing so, an average service life of 1000 years is an over-estimate because the annual replacement rate of a housing stock declines as the expansion rate of the housing stock increases. The annual replacement rate of a housing stock is a function of the distribution of dwelling losses from each dwelling cohort over its service life span, the distribution of dwelling by age, and the expansion rate of the housing stock.

Meikle and Connaughton [23] state that a low level of stock replacement means that the housing stock is ageing. A decline in the annual replacement rate is an indicator that a stationary housing stock is ageing but is not an indicator that an expanding housing stock is ageing unless the expansion rate of the housing stock also declines. The mean age and replacement rate of a housing stock both decrease when the expansion rate of a housing stock increases and the mortality of the housing stock remains unchanged.

Meikle and Connaughton [23] conclude that if household formation in England did not increase, then it would take 100 years to replace the current stock, assuming net gains of 200,000 per annum (and a total stock of approximately 20 million dwellings). If the mortality regime of housing stock in England should remain unchanged in the future, then the period

required to totally replace the current stock of dwellings would be the current service life span which is at least 273 years.

The average service life and service life span of a housing stock are indicators of mortality and are not indicators of the quality of dwelling services provided by a housing stock. Indicators of the quality of dwelling services include the mean effective age of the housing stock and the service loss index or mean depreciation of dwelling services. The average service life and service life span of a housing stock may increase due to an increase in the extent and number of cycles of rehabilitation which reverse the depreciation of dwelling services, extend the life expectancy of dwellings, and hence reduce the annual replacement rate of the housing stock. Alternatively, the average service life and service life span of a housing stock may increase due to a reduction in the annual replacement rate which is not accompanied by an increase in the extent and number of cycles of rehabilitation. The quality of dwelling services provided by the housing stock would subsequently decline. Change in the ageing of a housing stock due solely to changes in the expansion rate would be gradual, as there are time lags inherent in the redistribution of dwelling cohorts by age. A decline in the quality of dwelling services provided by a housing stock would be rapid in comparison if current levels of maintenance and rehabilitation were to immediately diminish.

7. Conclusions

The mortality of a housing stock can be established empirically by either developing a life table model of the housing stock which makes use of age-specific data on dwelling losses and survivorship or, in the absence of suitable data, a stock and flow model of mortality which is validated against available data. The realism and subsequent precision of each model is limited by the implicit assumptions of mortality built into the model.

The ideal life table model is a generation life table based on longitudinal data. A series of generation life tables enable detection of dynamic mortality and general shifts in mortality over time. In practice, it is unlikely that any country has compiled a series of statistics on age-specific dwelling losses and survivorship over the service life span of housing stock. Until such data series are available, researchers of housing stock dynamics are restricted to using current life table models that make use of cross-sectional data or stock and flow models of mortality.

Use of a current life table requires adoption of the implicit assumption that the mortality of a housing stock is static. A current life table approach cannot

offer precision in estimating the true mortality of dwellings should the housing stock be subject to dynamic mortality. A stock and flow model of mortality can establish whether a housing stock is subject to static or dynamic mortality, but lacks the potential to estimate the mortality of a dwelling cohort to the same precision as that of a generation life table which makes direct use of age-specific data on mortality.

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